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AN

ELEMENTARY HANDBOOK

OF

THEORETICAL MECHANICS,

WITH 145 DIAGRAMS.

BY

WILLIAM ROSSITER,

F.R.A.S., F.C.S., F.R.G.S.

LONDON AND GLASGOW:
WILLIAM COLLINS, SONS, AND COMPANY.
1873.

I OFFER THIS BOOK

TO

ARTHUR HILL, Esq.,

OF BRUCE CASTLE, TOTTENHAM,

IN

GRATEFUL ACKNOWLEDGMENT THAT I OWE MUCH TO HIS TEACHING AND FRIENDSHIP.

W. R.

LUCERNE, *September*, 1872.

PREFACE.

THE main purpose of this book is to explain, in simple language, so much of the Science of Theoretical Mechanics as is required for the Elementary Stage of the Science and Art Department's Examination in that subject.

But the book has not been rigidly confined to the Syllabus of that Examination, though, for convenience of reference, the order of it has been followed in the earlier part.

In the diagrams my chief aim has been to illustrate the text; they are therefore very simple in character.

W. R.

WORKING MEN'S COLLEGE,
BLACKFRIARS' ROAD,
LONDON, *January, 1873.*

CONTENTS.

THEORETICAL MECHANICS.

1. Measurement of Time.—If all the clocks were to be stopped, how could we tell the time, so as to be able to set them going again? and how could we tell if they were going correctly?

The sun being at its highest point in the sky on any day, is taken as being noon, or twelve o'clock, at that place—that is, it is always twelve o'clock (noon) below the sun; and thus, noontime travels round the earth from east to west with the sun. The time in Cornwall is about a quarter of an hour behind the time in London: these fifteen minutes being the time the sun takes in getting from London to the Land's End—which gives a rough notion of the rate of travel of the earth round the sun. If I make a clock-face with a hand to turn once round it between two visits of the sun, I have the measure of a *day* (this being the name given to that lapse of time), and can divide the interval how I please, and mark the divisions on the dial; then I can divide it into ten, twenty, fifty, or any number of subdivisions. Practically, our clocks are measurers of only half a day, and this half day we divide into twelve equal parts, called *hours;* these again we divide into sixty equal parts, called minutes; and these again into sixty seconds.

It might seem that the first moment of the sun's appearance in the morning might be a better time to take than that of his greatest altitude, which is not so distinctly marked. But, if this were taken, some authentic standard would have to be set up by some competent observer; and this is much easier in the case of the sun's highest rise than his first appearance, which is more difficult to observe accurately, owing to the refraction of the air, in addition to which the rising and setting are not recurrent at regular intervals. Neither, in fact, does the greatest altitude occur regularly at intervals of twenty-four hours: the variation is sometimes several minutes; but the average of the whole year is taken. Also, there is about a quarter of a day at the end of each year, over and above the 365 days; but this is very nearly rectified by making every fourth year 366 days. So that, finally, we may say that in every four years there are 1461 days—that is, the sun rises and sets 1461 times. The whole four years is divided equally into 1461 equal intervals, each called a day; and the sun rises and sets, in most parts of the earth, once in every one of these intervals, the noon of which is marked by the average time of the sun's highest rise.

So that to define a second of time, we are compelled to consider the time of the revolution of the earth on its axis, corresponding nearly to a day, and the revolution of the earth round the sun, which corresponds nearly to a year.

Thus:—A second is the $\frac{1}{60}$ part of a minute.

A minute is the $\frac{1}{60}$ part of an hour.

An hour is the $\frac{1}{24}$ part of a day.

A day is the average time between two appearances of the sun at its highest point in the sky.

Also, a day is $\frac{1}{1461}$ part of the time occupied by the earth in going four times round the sun.

2. Measurement of Space.—If all our yard measures

were broken or lost, how could we replace them? Practically, by reference to the standard yard measure, kept (I believe) in the Tower of London. But this would have to be measured at a temperature of 60° Fahr.; for, if it were warmer, it would be too long, and if colder, too short. But how was this determined? A yard is the length of a pendulum, which, with a bob of a certain size and weight, vibrates regularly once a second. It may seem odd that the vibration of a pendulum should be taken as a means of determining a measure of length, but it is really one of the most delicate tests of length.

The French unit of length is a metre, a little longer (nearly $\frac{1}{10}$) than an English yard, and this is determined as being $\frac{1}{40000000}$ of the circumference of the globe measured round the poles.

The English yard is subdivided into feet, and the feet into inches. The French metre is subdivided into decimetres and centimetres.

English.	French.
1 yard = 3 feet.	1 metre = 10 decimetres.
1 foot = 12 inches.	1 decimetre = 10 centimetres.

1 English yard = ·915 metre.
1 French metre = 1·094 yards = 39·384 inches.

3. Velocity.—If a thing move from one place to another, it may be necessary to describe the rate of its movement as compared with some other rate. To say that one body moves more quickly than another is too vague and indefinite. Therefore, motion is usually described in terms of distance and of time. To say that a ball has rolled ten yards, or that it has been moving for ten seconds, gives no idea whatever of its rate of motion; but to say that it has been rolling for ten seconds, and has moved ten yards in that time, gives a complete idea of its velocity: one yard per second. It is necessary, therefore, but also it is sufficient, to express rate of movement, or velocity, by saying what distance has been traversed in a given

time. In English books, seconds and feet are the measurements of time and space most generally used: in French, metres and seconds are used.

That is to say, the second is the usual *unit of time*, the foot (or the metre) the usual *unit of space*.

4. Uniform Velocity.—If a ball has moved through ten feet of space in ten seconds, and has also moved through one foot in each second, its velocity is said to have been *uniform* or regular; it may have been irregular, since it may have traversed one foot in each second, and yet varied in rate during each second. But if it has traversed half the given space in half the given time, one quarter of the space in one quarter of the time, one tenth of the space in one tenth of the time, &c., then its rate of motion has evidently been uniform and unchanged. Instances of uniform velocity are the motion of a railway train, in which the force of the engine just counterbalances the impediment of friction; the motion of a carriage, of which the force exerted by the horses does the same; the raising or lowering of a bucket at a well by means of a rope wound or unwound round an axis turned with unvarying speed.

5. Accelerated and Retarded Velocity.—I roll a smooth ball along a table or the floor: its motion becomes slower and slower until it finally ceases to move, the force originally imparted to it having been gradually counteracted by friction. Here the velocity is said to be *retarded.*

I drop the same ball from the top of a house, or a tower, and it falls to the ground with a continually increasing rate of motion. When I let it free from my hand, gravitation acts upon it, and it begins to fall; but gravitation continues unceasingly to act upon it, and consequently it falls more and more rapidly. Just in the same manner a train, of which the engine exerts more force than the friction counteracts, moves with a constantly increasing speed. In such cases the velocity is said to be *accelerated.*

Velocity may be accelerated either continuously or at

intervals. In the examples of a falling ball and a train moved as above described, the acceleration is continuous, and the velocity is constantly increasing. If a boy bowl a hoop, and by repeated blows make it move more and more rapidly, the increase of velocity is intermittent: the hoop passes through a greater space every second; but between any two blows its rate of motion diminishes from friction, although, since the force applied more than counterbalances the friction, the average speed is continually increasing.

6. Quantity of Matter.—If I weigh a pint of water, a pint of milk, and a pint of spirits of wine, I find that the volumes are not equal. So, if I weigh equal volumes of lead, cork, and zinc, I find the weights to be different. This is explained by the supposition that in the heavier bodies the particles are packed more closely than in the lighter. The earth is supposed to exert the same attractive force upon each atom; and, therefore, if the atoms be more closely packed in one body than in another, the same power of attraction is exerted upon a smaller volume. So that we may say either that equal weights occupy unequal volumes, or that equal volumes have unequal weights.

I take a pound of lead and a pound of cork—the cork is much larger than the lead. Here we have equal weights and (according to one theory) equal quantities of matter. In one, the atoms are more compact; and, therefore, we have the same attractive force exerted on a smaller body. I add to the pound of lead until its volume equals that of the cork. Now, I have equal volumes but unequal weights. The atoms of one being more compact than those of the other, I have unequal attraction for equal volumes.

It must be borne in mind, that what we call weight is the exertion of an attractive force by the earth. When I say that a piece of iron weighs a pound, I mean that the earth pulls it downwards with a certain force. The weight of a substance is not something belonging to it nor any force exerted by it, but a force

exerted on it by the earth. This force is exerted upon matter in its constituent particles, without reference to their number, or to the way in which they are grouped.

7. Momentum.—A small stone falls from a window near the ground, and is scarcely felt. Had it fallen from a higher window it would have fallen with increased force: had it been larger, its effect would have also been increased. In the one case it would have exerted more force, because its velocity was greater; in the other, because the quantity of matter was greater.

If a stone be thrown against a window without breaking it, it may, by being thrown with greater force, be made to move with greater velocity, and so to break the glass; or the same effect may be produced by throwing a larger stone with the same velocity. In either case, a greater effective force is brought into action; and such effective force is spoken of as the *momentum* of the moving body.

The amount of force which a moving body is capable of imparting to any movable body which it comes in contact with, is called its *momentum*, or effective force. Thus, I roll one marble against another: the first parts with its force to the second and comes to rest, while the second moves.

8. Force.—Just as water can be conveyed from one place to another in a suitable vessel, so can force be transmitted: thus, a stone falling from the top of a house would break the glass of a summer house, if it fell on it. But really it is gravitation that causes it to break the glass; for the stone might be laid on the glass without breaking it, though, when drawn down by gravitation, it passes through.

The stone could not fall except it had been at the top of the house. How did it get there? It might have been carried up years ago, when the house was built; or but a moment before it was let fall. But whenever it was carried up, *then* it received the force that enabled

it to break the glass. The labour of carrying up the stone was what gave it the power to fall; the higher it were carried the greater would be its force in falling; the more labour spent in raising it, the greater its force in falling.

Force is in no sense inherent in any substance. A stone thrown against a wall falls and lies on the ground as harmless as before. The force of the blow against the wall was, as it were, lent to the stone by the strength of the throw:—For example, a cricket-ball struck by a bat passes through a window, against which it strikes by accident. The window receives force from the ball, which does not go so far as it otherwise would; the ball receives force from the bat, which, had it missed the ball, would have been thrown much farther, instead of being stopped by the ball. The bat receives force from the movement of the arm. So that we may say, that the force exerted by the arm is transferred by means of the bat and the ball to the window, and that the ball conveys this force over a considerable interval of space.

Since force cannot exist inherently in any lifeless body, where are we to look for it? At first, it might seem enough to say that it is only to be called into action by human, or at least vital energy; that without life on the earth there could be no force. But, though it be true that wherever there is life there is the power of exerting force, it is not true that force exists only where life is. In the storm of wind or rain, in the lightning, even in the lightest breeze, there is force, though no living thing be near. But all these owe their origin to the movement of the earth through space; and this is due to causes which we have no power to comprehend or describe.

9. Absolute Unit of Force.—Force being defined as something capable of producing motion, how are we to calculate it? To say that a man is a strong man gives no adequate idea of his power, since what one man calls strong, another may consider weak. In speaking of

length, we have a foot or a yard as a unit; in speaking of time, we have a second as a unit, and of weight, a pound. What shall be the unit of force? Evidently the power of moving some given weight some given distance. The ordinary units of weight and of distance are evidently the most convenient; and so to raise one pound through a distance of one foot shall be the task for one unit of force.

Evidently, it is the same whether ten pounds be raised one foot, or one pound be raised ten feet, or two pounds be raised five feet, or five pounds two feet. So that any given force may be measured by the amount of motion it has the power of causing—the unit of measurement being the movement of one pound through one foot of distance against gravitation.

10. Elements Required for the Specification of a Force.—I strike a ball when playing at cricket, or discharge an arrow from a bow, and I wish to describe the result to one who is not present. He will want to be told two things: how far the ball or arrow went, and in what direction? The distance by itself will not be sufficient for an accurate description: nor will the direction by itself suffice. I must tell him how far it went, and in what direction: he then has sufficient data. And so of any force: it is essential, but also sufficient, to describe the direction and the extent of the motion that ensues, or that would ensue if there were no counteracting force. Thus, I express the effect of my blow upon the cricket-ball by saying it went so many feet; if I add in what direction it went, all is known that can be known as to the force applied to it. Of course, it is also necessary to say, where the ball started from—*i. e.*, the point of space where the force was applied. But this last is only necessary for the calculation of some definite problem, not for the description of the force applied.

This may be reduced to saying that any force may be described by saying of how many units of force it consists; adding the line of action for the purpose of

calculation with other forces, and the point of application in the case of a definite problem.

11. Composition of Forces.—When I strike a ball, it will, if free, move in a certain direction to a certain distance. I can easily denote this direction and distance by a straight line, such as A B, the length of which can be increased or decreased as the force is greater or less. Thus, a force of 1 ℔

Fig. 1.

may be shown by a straight line 1 inch, 1 foot, ·1 inch, or any other length. Then a force of 2 ℔s, which would send a ball twice as far as the force of 1 ℔, may be shown by a line of 2 inches, 2 feet, ·2 inch, &c. The *direction* in which the force acts may be shown by the order of the letters, or by some other method.

This method is useful in many ways, but especially in showing the connection between several forces acting together. Thus, supposing a ball to be struck two blows at once, the blows acting in different directions with different forces. This might be shown thus, the point A marking the original position of the ball, and A B, A C, showing the directions of the two forces acting on it. They also show the relative strength of the blows. If the blow acting along A C be three times the strength, of the blow acting along A B then the line A C is three times the length of A B, and so on.

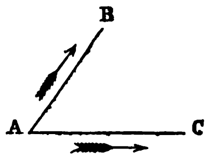


Fig. 2.

So that by means of straight lines we show very accurately the direction and amount of forces, and also their relation to each other. If necessary, arrows can be used to show the direction. If I strike

Fig. 3.

a ball with a certain force, in a certain direction, it goes

a certain distance. This I mark by a line A B, which shows both the direction and extent of the consequent motion of the ball. If now I strike the ball two blows at the same moment, I show this by two lines, A B, A C, each line showing the amount and direction of one of the forces acting by itself. That is, if the force along A B acted by itself, the ball would travel from A to B. If the other force acted by itself, the ball would travel along A C to C.

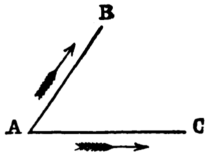


Fig. 4.

But when both forces act at the same time, the question becomes:—Since the ball cannot travel along A B, because of the force acting towards C, nor yet along A C, because of the force B, what direction will it take? and what will be the extent of its motion?

We may safely assume that the whole of each force will be used up in motion, since nothing can destroy it. Let us consider each blow separately, taking B first. That will send the ball along A B to B. Now, suppose the second blow to act. To show this we must draw B D equal to A C, to show the amount of the force, and parallel to A C, to show (as nearly as we can) its direction. Then the ball, by the action of the second force will be sent along B D to D. So that, reckoning each force to exert its full effect in moving the ball, its path would be along A B to B, and then along B D to D.

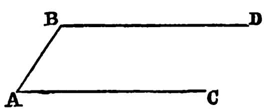


Fig. 5.

Let us now see the effect of reckoning the forces in the reverse order; first the force A C, and then the force A B. The first will move the ball from A to C, along A C. The action of the second, if reckoned to

begin at C, will be along C D to D, C D being equal and parallel to A B. So that, according to this estimation, the ball will move from A to C, and from C to D.

Fig. 6.

It is evident that D is the same point in either method of estimation, since A B C D is a parallelogram, and I arrive at the same point, D, whether I go to it from the A through B, or from A through C. In the two cases the lengths are equal, the directions parallel, the results identical. This being the case, the question becomes:—Admitting D to be the point reached, which is the road by which it is reached? Since the two roads are equal and symmetrical, what is to decide which shall be taken? The answer is:—Neither A B D nor A C D is the road by which the ball reaches D. When two weights are exactly equal, they balance, and neither moves. So the ball, urged by two equal impulses, takes the middle road, A D, the diagonal of the parallelogram, A D.

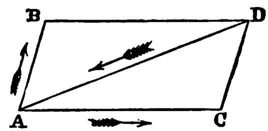


Fig. 7.

When, therefore, we have two forces, the amounts and directions of which we know, we find the extent and direction of the actual resulting motion by drawing two lines, A B, B C, to represent the two forces in amount and direction, completing the parallelogram by drawing B D and C D, and drawing the diagonal A D. This diagonal shows the direction and extent of the resulting motion, and D shows the point reached. So that a ball at A would be moved along the line A D, through the distance A D, by two forces acting, one along A B and the other along A C. But any accurate thinker may suggest, this is not a problem in STATICS, but in DYNAMICS. The science

of Statics is the science of rest, not of motion. We do not want to know where and by what road the ball will move, but what will prevent its motion. To this I can only say, that to know how to prevent motion, it is necessary to know in what direction it will tend to be, and what force will tend to cause it. Finding the extent and direction of the actual resulting motion, A D, is the best means of ascertaining the extent and direction of the force necessary to prevent it. Thus, knowing that the two forces, A B, A C, will, by acting together, produce motion along the line A D, I also know that if I apply a second and equal force in the reverse direction, D A, I shall prevent the ball moving at all.

The problem in Statics is:—Given, two forces, the extent and direction of which are known, acting upon a body at a given point; required, the extent and direction of the force competent to prevent these forces producing any motion. The solution of this problem is:—Construct, as before, the parallelogram A B C D, then the diagonal A D shows the direction and extent of the actual resulting motion; also the same diagonal *reversed* —*i.e.*, D A, shows the direction and also the extent of the force necessary to prevent this motion resulting. The diagonal A D is the answer to the dynamical question: What motion will result? The diagonal D A is the answer to the statical question: What force will prevent motion?

12. The Triangle of Forces.—But I imagine I hear some utilitarian reader saying: Why take the trouble to draw four sides of a parallelogram, when two are all you require? Given, A B and B D; you can find A D without drawing A C or C D.

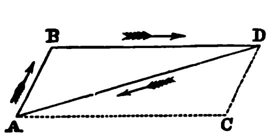


Fig. 8.

This thought worked out will give us an easier and quicker method of finding the answer to our question:

What is the extent and direction of the force necessary to prevent motion resulting from the application of two given forces?

Let there be two forces, A and B, acting at A, as before. Instead of drawing A B and A C both from A, draw A B and B D: A B to show the extent and direction of A, and B D to show the extent of B, and drawn parallel to its real direction, A C. Then the side A D, completing the triangle, will give the extent and direction of the resultant force, and D A will give the extent and direction of the force necessary to prevent motion. This is a simpler construction than the one requiring the parallelogram to be completed; but the results are identical. The only difference is, that the lines representing the forces are drawn so that they form a triangle with the line representing the resultant.

This construction is called the "TRIANGLE OF FORCES," because it shows that any three forces that can be represented by the sides of a triangle will result in equilibrium, so that the body acted on by them will remain at rest. This requires, however, that the *directions* of the forces shall form a set of continuous lines—that is, from A to B, from B to D, and from D to A, as shown by the arrows. If either of these forces have its direction reversed, this is no longer true.

The "Triangle of Forces" also shows that if any two forces be represented by two sides of a triangle, the line joining these two, so as to make the third side of the triangle, will represent the resulting motion, and also the force that will prevent it. But if there be more than two forces—say, three or four—what modification of our theorem is necessary? We cannot have a four or five-sided triangle, nor a five-sided parallelogram. We shall, however, find our theorem elastic enough to comprehend not only three or four, but any number of forces.

Let us consider first the extension necessary in the case of the parallelogram. Let there be, as before, two forces, A B and A C, and also a third one, A E. To

show these we draw the three lines, A B, A C, and A E. Taking two of these—say A B and A C first—we construct the parallelogram A B D C, and the diagonal A D, gives, as before, the resultant of these two forces. Now, taking this resultant, A D, and the third force, A E, as being two forces, we construct a second parallelogram, A D F E, and then the diagonal of this, A F, is the resultant of these two, and consequently of the whole three. We can always treat the resultant of a number of forces as representing the whole of them, and speak of A D as the two forces A B and A C, also of the resultant, A F, as being the three forces A B, A C, and A E.

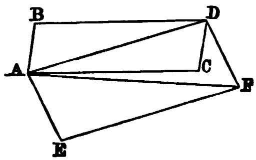


Fig. 9.

Thus, if four forces, represented by A B, A C, A D, and A E, act upon a ball at A, we find A F the resultant of A B and A C; then, in same way, A G the resultant of A D and A E. Treating these two resultants as two forces, we find their resultant A H, which is the resultant of the whole four forces, A B, A C, A D, and A E.

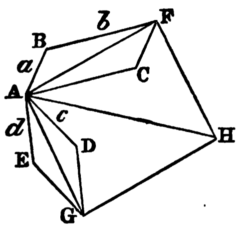


Fig. 10.

This same method may be extended to any number of forces acting at a point. The final resultant A H shows the extent and direction of the resulting motion, and H A, the reverse direction, shows the extent and direction of the single force that will prevent motion resulting from the action of the others.

This is an extension of the theorem called the "parallelogram of forces." But we saw that we could simplify this "parallelogram" into a "triangle of forces."

Can we in any similar way extend the "triangle" method so as to use it for any number of forces? The distinctive principle here is to consider each force as acting separately, and drawing each successive line from the termination of the preceding one, instead of from the actual position of the ball. Let us apply this to the case of the four forces in the problem on page 22, the figure of which is given here again for comparison, and to show the superiority of the simpler method in point of construction, though it does not show the reason of the line A E so well. But it is evident that the resultant force is really and accurately found by either method, both as to amount and to direction. The line A B shows the amount of the force *a*, in either figure; the line B C (corresponding to the line A C in fig. 11) shows the amount of the force *b*; the line C D shows the amount of *c* (corresponding to the line A D in fig. 11); and the line D E (corresponding to A E) shows the amount of force *d*. In fig. 11 all the force-lines are drawn showing both *amount* and *direction*. In fig. 12 all the force-lines show the *amount*, but *not the direction*, being each drawn *parallel* to its line of action. Each force is *considered* to act from the point where the preceding force left the ball. Fig. 11 shows an extension of the simplest case of the "parallelogram of force," of which it is itself an example. But fig. 12, although showing an extension of the "triangle of force," cannot be considered to be an example of it, because the

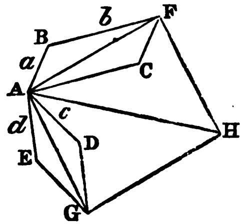


Fig. 11.

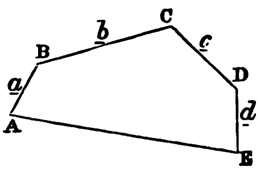


Fig. 12.

figure A B C D E is not a triangle but a polygon. We must therefore find a more comprehensive name. The term *triangle* was given, in the case of *two* forces, because the two force-lines, with that showing the resultant, made up the three sides of a triangle. So that the term "triangle" does not describe the principle *generally*, but only in the case of *two* forces. This is true also of the parallelogram. But the term "POLYGON OF FORCES" will comprehend all possible cases, of any number of forces.

Just as we make, by one method, a series of parallelograms, each giving the result of two forces (each of which may itself be a resultant), so, by the other method, we can construct a series of triangles. Thus, from A to C, I draw the line A C, representing the resultant of the two forces, *a* and *b*. In the same way, I can draw the line A D which represents the resultant of *a*, *b*, and *c*, and also make a triangle, A C D, with the lines A C and C D.

13. Resolution of Forces.—We are now familiar with the methods by which we can find the practical result of any number of forces acting on any body at a given point. To the problem: Given, any set of forces acting simultaneously upon a ball, find the extent and direction of its motion: we can supply an accurate solution.

But there is another phase of this problem, the reverse question: *Given, that a ball moves a certain distance in a certain direction, to what force or forces is this motion due?*

If we are asked the sum of 9 and 16, the answer is 25, and no other answer is correct. But if we are asked, What two numbers make up 25? the answers are infinite in number. It may be $12 + 13$, $10 + 15$, $9 + 16$, $7 + 18$, and so on.

So when we are asked, What forces will produce any given amount of motion? the number of answers is infinite. But if we are given one force, we can find the other, if there be two; if three, and we have two given, we can find the third; and generally, if there be given any number of forces, we can find the resultant: or, *vice versa*, if we have given us the resultant and all the forces but one, we can find that one; if the resultant and all the

forces but two, then we can find the resultant of those two.

This we may do by means of the "parallelogram of forces," or by means of the "polygon of forces," the latter method being the more simple and elegant. Let us take an example of the former method.

Fig. 13.

We have given us the resultant, A D, and one force, A B, to find the other. We join B D, and complete the parallelogram A B D C; then A C represents the extent and direction of the second force; that with the given force A B has the given resultant A D.

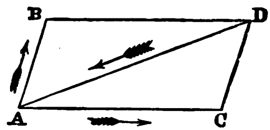


Fig. 14.

By the "triangle of forces" the method is: join B D, as before; this shows the *extent* of the second force. A line equal to this, and parallel to B D, but drawn from A, shows the direction.

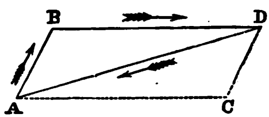


Fig. 15.

It will, however, be noticed that the *same resultant* may be produced by *any* number of forces.

Thus, the first resultant, A H, may be produced by the two forces, A F, A G; or by the three forces, A B, A C, A G; by the three, A F, A D, A E; or by the four, A B, A C, A D, A E: so that the question, What forces are requisite to produce a given resultant? is not capable of a simple reply.

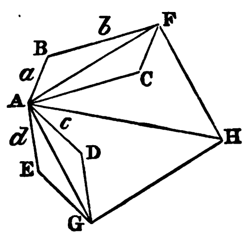


Fig. 16.

A very simple construction, however, will show the relation between any given motion and the forces that will produce it. For, let A D show the extent and direction

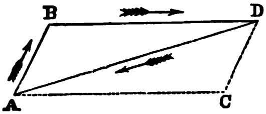


Fig. 17.

of any motion; then the body so moving may be acted on by a *single* force in the line A D, acting at A; or by two forces acting at A, one equal to A B, and the other to B D. The line A C shows the real line of action of B D.

But A B D is a triangle; and we come to the fact that if any line whatever shows the extent and direction of a resulting motion, that resultant may be produced by *any two lines* which, with the resultant, form a triangle. These two lines show the *extent*, and, if they be imagined to act at A, also show the direction of the forces producing the given result. But

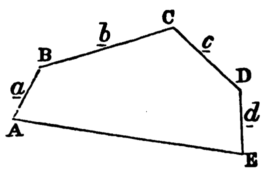


Fig. 18.

each of the lines so joining the triangle may be itself resolved into two, or any number, of other forces. Thus, if A E be the resultant, and I make the triangle A D E, then the forces A D, D E, are competent to produce the resultant A E. But A D may be itself produced by two forces, A C, C D, which make a triangle with A D. In the same way the force A C may be itself the resultant of other forces, A B, B C.

Speaking generally, therefore, we may say that a force may be the resultant of *any number* of forces, and that the *extent* and *direction* of a group of any given number competent to produce this resultant may be found by drawing as many lines as there are forces, so as to form with the resultant line a polygon.

14. Composition of Parallel Forces.—We have seen

that two forces acting together on any given particle produce, or tend to produce, motion in a straight line, which might also have been produced by one force. In fact, when a particle is seen in motion, it is impossible to tell by its motion how many forces have concurred to produce it. We have also seen that the resultant of any two forces may be found by making a parallelogram or a triangle, having the lines representing the forces for two adjacent sides.

But supposing the two forces to be parallel, as in the case of two horses drawing a carriage, how can we make either a triangle or a parallelogram in this way? There are two ways of getting over the difficulty: first, by calculating according to another method; secondly, by an artifice whereby a triangle or a parallelogram can be formed, but which need not be considered here.

To take the first method, the resultant of two forces is a single force equivalent to their combined action, acting along the same line of motion. A single force equal to this and acting in the contrary direction would prevent motion. If, therefore, we can find the point where a single force will prevent motion resulting from two parallel forces, we have the point of application of the resultant.

If A D and B C be two parallel forces, then E F will represent the resultant, and the point E will be found by dividing the whole distance A B into two parts, A E and E B., so that A E : E B : : B C : A D. That is, the single force is to act nearer to the greater of the two forces than to the lesser. If A D and B C be equal, then E is the middle point of A B.

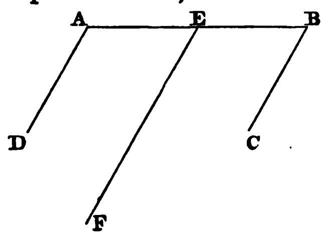


Fig. 19.

Since A D and B C are parallel, all the force of each

is effective, and therefore E F is equal in length to A D + B C; also, since none of either force is lost or counteracted, the resulting motion is parallel to their direction.

So that E F is parallel to A D and B C; also, it is equal to A D + B C, and its point of application is at E, when A E : E B : : B C : A D.

It will be seen that if the two parallel forces were unequal, and a force equal to them both were applied at the middle point of A B, then the greater force D would tend to move round the point A in a circle. Thus, if two horses of unequal strength were drawing a carriage, and a force equal to their combined strength were applied at the middle point between them, the weaker horse would be altogether stopped, while the stronger would still have a little power; and the result would be a circular motion of that part of the carriage to which he was fastened round the point of application of the resisting force. If, however, the resisting force were applied nearer to the stronger than to the weaker, both might be completely stopped.

If, however, the points A and B coincide, the two lines A D and B C also coincide with each other and with the resultant E F. That is, if the same particle be struck at the same moment two blows, each tending to move it in the same direction, it will move in that line with the combined force of both. For example, I strike a ball with a force tending to move it 50 feet, and at the same moment it is struck also by some one else with a force tending to move it 60 feet in the same direction. It will move $50 + 60 = 110$ feet.

If the two blows had tended to move the ball in opposite directions, then it would move $60 - 50 = 10$ feet—*i.e.*, equal forces in opposite directions counteract each other.

15. A Couple.—If a stick be struck at the same moment at both ends with equal forces, but in opposite

directions, the result will be to turn the stick round its centre of gravity. The resultant of these two forces might be said to be applied at the centre, but in what direction? It will be found that in this case the resultant force cannot be expressed by a single straight line; for no single force can be applied so as to keep the stick from moving. According to our previous method,

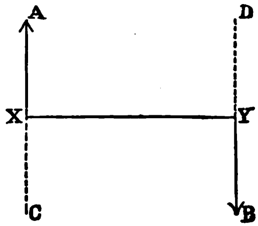


Fig. 20.

the resultant would have its point of application at the centre of the stick; but this is just the one point that does not move. To counteract the action of the two parallel equal forces A and B, acting in opposite directions at the extremities of the stick X Y, we must apply two other parallel, equal, and opposite forces, C and D, so that these two pairs shall counteract each other. Such a pair of forces, equal, parallel, and opposite, is called a *couple:* it is evident that one couple can be applied so as to counteract another, provided their effective forces be equal. Two such couples are called *unlike,* if their resultants tend to cause motion in opposite directions; and *like,* if their resultants tend to help each other.

16. The Moment of a Force.—A force applied in one direction may produce motion in another: we have seen that a ball struck by two forces at the same moment travels in a line between the two; but some part of each force is lost. Thus, if one force would send the ball from A to B, ten yards, and another would send it from A to C, also

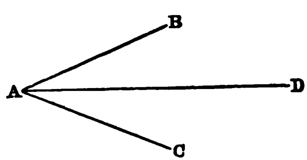


Fig. 21.

ten yards, it will travel along A D; but not for

twenty yards, unless, indeed, that A B and A C coincide.

Again, a ball weighing two pounds, hung at the end of a rod, exerts a force of two pounds towards moving the rod downwards. But if the string by which the weight is suspended pass over a peg at B, so that it does not act at right angles to A, then it does not exert its full weight towards depressing the stick at A. In all such cases the amount of effective force is a matter of calculation; but whatever it be, it is expressed by the name *moment* (abbreviation of momentum). So that the power of any given force to produce motion in a given direction is called the moment of that force in that direction.

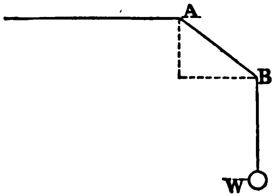


Fig. 22.

17. Centre of Parallel Forces.—A number of horses abreast of each other might be pulling a cart along a road. Here we have several parallel forces, and it is scarcely probable that they would be all exactly equal. Where would be the point of application of their resultant? That is, at what point could we apply a single force, equal to all the others, so that no motion should follow? Let A B C and D be four forces, acting side by side. Join A B, and take the point a, so that $A\,a : a\,B :: B : A$. If A and B be equal, this point a will be half-way between them; if A be the greater, a will be nearer to A; and if B be the greater, a will be nearer to B.

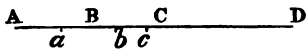


Fig. 23.

Then join a C, and take the point b, so that—

$$a\,b : b\,C :: C : A + B.$$

Since a is the centre of application of both A and B, b

is sure to be nearer to *a* than to C, unless C equals both A and B, in which case it would be half-way between *a* and C. Then join *b* and D, and take *c*, so that $b\,c : c\,D :: D : A+B+C$. Then the point *c* is the centre of application of the whole four forces, A B C and D. An equal and opposite force applied at *c* would prevent motion; or, if the four forces, A B C and D, were all acting at *c*, the same effect would be produced as by their being at the points A B C and D. So that it is the same whether the four horses pull side by side at the four points, A B C and D, or whether they all pull at the one point *c*.

What is true of four horses pulling a cart is true of any number of parallel forces, however applied and however derived. The point of application of any two parallel forces is between them, but nearer to the greater force. If there be more than two, the point of application is found by taking first two forces, and then the resultant of these with the third, and so on, as already described. In every case it will be found that equal force is applied on each side of the centre.

Compare this with the fulcrum of a lever kept at rest, and with the method of calculating the centre of gravity of a number of bodies connected.

18. Work Done by a Force.—We may want to estimate a force, or a number of forces, not in terms of the force or forces, but of the work done. All work is done by force; and we may speak of the work as the result of so much force, or of the force as the cause of so much work. We have also seen that to raise one pound weight through one foot of space is the unit of force; but in estimating the work of powerful machinery, pounds and feet are, by comparison, very small, and it is desirable to have some larger units. Just as in measuring long distances, we speak of miles and leagues rather than of yards or feet; and in the case of great weights we speak of hundredweights and tons rather than of pounds. But the miles

and leagues are merely larger units, each containing a known number of feet; and the hundredweights and tons are merely larger units, each containing a known number of pounds. The question now is, therefore, what larger units than a pound and a foot shall be used in estimating the work of powerful forces?

19. Unit of Work and Horse-power.—The power of raising 33,000 pounds weight through a distance of one foot is called a "horse-power." But even a mouse could do this *in time*—so that it is necessary to take time into consideration. In this way the work of raising 33,000 pounds weight through a space of one foot in one minute has become the definition of a "horse-power"—*i.e.*, a horse was supposed by Watt (by whom this particular number was calculated) to have strength enough to raise about 15 tons through one foot in one minute of time.

1 horse can raise vertically	550 lbs per second.
" " "	33,000 lbs per minute.
" " "	1,980,000 lbs per hour.

7 men could do about the same.

If we use this term "horse-power" as a means of comparing one force with another, we must keep time as an element in the expression: for, to say that one machine is of 100 horse-power and another of 200, means that one can do twice as much as the other in any given time.

But if we use the term merely as the expression of a quantity of work to be done, time need not be taken into account. If I wish to express merely that a certain mound of earth, or a stack of hay, weighs so much, I can say it is so many "horse-power," using the term merely as a convenient unit of weight.

20. Different Conditions of Matter—Ice.—I place a piece of ice in a ladle over a fire. It turns gradually to water, and finally to steam. Throughout, its nature remains precisely the same: it is always a compound

of hydrogen and oxygen. Its weight remains also unaltered. But there is this great difference between the ice and the water, the water and the steam: the ice is a *solid*—that is, its particles are closely united; the water is a *liquid*—that is, its particles are merely resting one on another, but are not otherwise joined; the steam is a *gas*—that is, its particles are altogether independent, and get as far as possible from each other.

The ice, when placed in the ladle, preserves its form unaltered; the water takes the form of the ladle below, but remains flat above; the steam rises above the ladle, and diffuses itself about the room. In the case of the solid, the cohesion of the particles prevents their spreading over the whole of the ladle: in the liquid this cohesion ceases to act, but the force of gravitation keeps the water in the ladle: in the gas the force of gravitation also ceases to act effectively, and the particles diffuse themselves into space. Conversely, having a vessel filled with steam, I cool it, and immediately gravitation, or the attraction of the earth, becomes effective, and I have a liquid, *water:* I cool it still further, so as to freeze it, and cohesion (or the mutual attraction of the particles) becomes effective, and I have a solid, *ice.*

What is true of water is true of *all* substances. Everything may be solid, or liquid, or gaseous. Which it shall be depends entirely upon the closeness of the atoms to each other: and this depends upon the degree of heat. If the particles are so close that mutual attraction acts between them, the substance becomes solid, and gravitation pulls it downwards as a whole. If they are just beyond this distance, but still very near, it is a liquid, and gravitation acts upon each atom separately. If they be still further apart, they seem to repel each other, and gravitation exerts but little if any power upon them.

At our usual temperature, water, mercury, and alcohol are always liquids; iron, lead, and copper always solids; oxygen and nitrogen always gases. But it is only a

question of heat, though we have no means of producing cold enough to freeze alcohol, or even to liquify oxygen or hydrogen. This is simply, that we have no power of producing the required temperatures.

21. Divisibility.—I take a piece of wood, and find I can cut it with a knife: the same is true of many other solids; for example, paper, meat, bread, cheese, &c. By cutting, I mean that I can separate the solid into smaller pieces. Every solid substance must be regarded as a number of small pieces held together by cohesion. I can overcome this cohesive force by breaking or by cutting: if by breaking, I get an irregular cleavage, since the parting takes place wherever the cohesion happens to be least powerful; if by cutting, I get regular cleavage, since the parting is made by the knife, without reference to the relative cohesion of the particles.

Cutting or breaking is no more than thus separating particles from one another; for all that is required to reproduce the original condition is to bring the particles together again as closely as before. Thus, if I cut in two a piece of soap or cheese with a clean, sharp, thin knife, and then press the pieces together again tightly, they become almost as firmly united as before. A piece of soft putty may be cut in two, and made again into one piece by pressure.

I may file a piece of metal into very small pieces, and recombine these into a single piece by melting. I cannot get them close enough together without melting, since I cannot get the air completely away from between the particles. Heat, however, does this, and allows the particles to come so closely together that cohesion is able to act as soon as the particles cool.

22. Compressibility.—I pour sand into a basin until it appears to be full: then, by pressing it tightly down, I can make room for more. The same is true of almost any other substance, probably of every other—that is, any given quantity of anything can be made to take up less space by having its particles more closely packed together.

If I put a piece of chalk in water, a number of bubbles of air rise through the water from the chalk; and if the chalk be taken out of the water and weighed, it will appear to be heavier than before. But gravitation cannot really exert any greater force upon it than before. Chalk is not a compact substance, having all its particles in the closest possible contact, but a group of particles, the interstices of which are filled by air. When put in water, this air is driven out, and its place filled by the water, which remains when the chalk is taken out. Thus, at first, I weigh chalk and air; afterwards, chalk and water; and water being heavier than air, the chalk appears to be heavier after immersion than before. Really, chalk and water are heavier than the same bulk of chalk and air. If I could compress the chalk so that all its particles were absolutely close together, with no intervals whatever between them, there could be neither air nor water, and the chalk would weigh exactly the same at all times; also, it could not be made to occupy any smaller space. If this were the case with all substances; if in all the particles were as close together as they possibly could be, the term "compression" would be altogether unknown, since the fact expressed by it—the compressing the atoms of a body more closely together—could not exist. But it does not appear probable that this is the case with any substance whatever. Water was long thought to be incompressible; but this has been shown to be an error. Every substance whatever, whether gaseous, liquid, or solid, must be considered to be a number of particles separated by small spaces, and capable of being brought into closer proximity by compression, if only sufficient force can be applied.

23. Elasticity.—A good table knife may be bent almost double, and yet, when released, will resume its original shape. A piece of copper wire may be bent in any direction, but will remain as bent without returning at all toward its first position. A piece of india-rubber, when stretched, returns nearly (if not quite) to its original dimensions. A piece of paper cannot be stretched. A

cork may be compressed or extended, and in either case tends, when released, to return to its first shape and size. A piece of putty may also be either compressed or extended, but remains in its new shape.

In every case the number of atoms and the weight remains unaltered: the only change is in the arrangement of the atoms amongst themselves; and the tendency to return to the original shape after alteration depends upon the force which any group of particles oppose to any alteration in their arrangement.

Thus, a piece of steel, A B, is bent in the form of a curve. This can be done only by the particles on the one side, A, being separated further apart than before, and the particles on the other side, B, being compressed. If the particles are so joined together as to be not easily deranged, we have elasticity—*i. e.*, they return to their original position as soon as released. It will be noticed that a piece of steel when bent too much snaps across just at the point of greatest tension. In this case the particles at that point are separated beyond the limit of their power to return to their first position, and part altogether. In other substances, such as clay, we have the particles easily separated and rejoined in any way—the cohesive power being but slight.

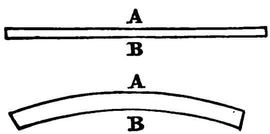


Fig. 24.

24. Gravitation.—I place a marble on a well polished and level mahogany table, or on a smooth marble slab. It remains just where I put it. I now tilt up slightly one end of the table or slab, and the marble immediately begins to roll towards the other and lower end—slowly at first, but with gradually increasing velocity, until it is stopped by some projection or rolls off on to the floor.

Four questions suggest themselves at once to any thoughtful observer:—

(1.) Why should it remain at rest until the table be tilted?

(2.) Why should it roll when the table is tilted?

(3.) Why should the velocity of the marble's motion increase?

(4.) How could the rolling be prevented?

1. *Why should it remain at rest until the table be tilted?* If I raise it a little from the table, it falls immediately when released. If I make a little door in the table, so that I can open it, just below the marble, it falls through when the door is opened. If, instead of a marble, I drop an open pen-knife, blade downwards, the point penetrates the table, more or less.

So that it would seem as if some unseen power were drawing the marble down, tending to draw it even through the table. A stone thrown from the top of a house falls to the earth. A ball thrown up falls again to the ground. In fine, it will be seen, on consideration, that everything is resting on the ground, with more or less tendency to sink into it. For even if I hold a stone in my hand so as to prevent it falling, yet it rests on my hand, and I rest on the ground. Even if I stood on a chair, the ground supports the chair; if the chair be in the highest room of the highest house, still it rests on the floor, the floor on the joists, these on the walls, the walls on the ground.

Therefore we may say that *everything rests, or tends to rest, on the ground,* with the exception of gases. To the ball resting on the table, the table is practically the ground, being supported by it. Our question, therefore, resolves into another, *What is it that draws everything to the earth?* To this the answer is, GRAVITATION; but when we are asked, What is Gravitation? we reply, We do not know. We say "Gravitation" glibly, and often fancy we have given a satisfactory reply. But it may be asked, "If you do not know what gravitation is, why use the word at all?" If gravitation be an unmeaning word, why use it as though it meant something? If it have a meaning, what is it?

I can only reply that, finding everything does descend to the ground—does gravitate to that as a centre—the word "Gravitation" is used to *express*, not to *explain*, this fact. It is convenient to have a word to express any phase of force which is constant in action; and in this sense gravitation is simply a name for a set of effects which (so far as human experience goes) is in continuous and regular action, just as are the terms Heat, Light, Electricity, &c., of no one of which can we give any adequate explanation.

To the questions, "What keeps the marble on the table?" "What draws everything to the earth?" we reply, "Gravitation," which, being freely translated, means, "Everything *is* drawn to the earth."

2. *Why should the marble roll when the table is tilted?* The answer to this is the same as to the previous question. When the table is horizontal, as A C, the marble cannot descend nearer to the earth, because every part of the surface on which it rests is equi-distant from the ground. But if the table be tilted, as A B, then the marble instantly begins to descend towards A, because in so doing it comes nearer to the earth—in fact, obeys the law of gravitation. If there be any ledge at A, then the marble will be brought to rest there; but otherwise it will travel toward A with increasing velocity and fall over the edge to the floor.

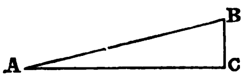


Fig. 25.

The power that sets the marble in motion down the inclined plane of the table is precisely the same that causes rivers to descend hill-sides, makes it easier to walk down hill than up, keeps rivers in their beds, ourselves from falling off the earth, and the earth itself in its place.

But it would be a mistake to suppose that "Gravitation" does no more than draw all things to the earth. Its power is much more extended, and its application very much more general.

To rise from small to great: Gravitation keeps the marble on the table: it also keeps all the planets in their orbits. To descend from small to smaller: we are told that even two specks of dust attract each other, as the earth attracts a marble, the only difference being one of degree.

We are familiar with gravitation only as shown between the earth and objects grouped on its surface. We see a stone fall to the ground, and we say "Gravitation;" we do not see any effect between two stones, however large, and therefore we are not conscious of there being the same attraction in and between them as exists between the great mass of the earth and both. The theory of the action of gravitation is, that two stones placed near each other in a complete vacuum, and away from *all* external influence, would immediately approach each other, just as the stone approaches the earth. But we could not find any place where we could place two stones free from the attraction of the earth; and therefore it is that we are so unconscious of the effects of gravitation other than the attraction of comparatively small objects to the earth. The power of the earth so far overpowers all minor attractions as to make them practically non-existent.

3. *Why should the velocity of the motion increase?* See page 56.

4. *How shall I prevent the motion?* By placing some projection on the table, so that the marble shall rest on it. Gravitation impels the marble one way—the ledge prevents it from moving. The result is that the marble remains at rest. This brings us to the problem of "Statics," *What force is required, and how shall it be applied, to prevent any given force, or set of forces, from producing motion?*

When a grocer weighs a pound of sugar, he works out a problem in Statics, How much sugar, acting in one scale, will prevent a pound weight, acting in the other, from moving the beam? This is a simple specimen of the problems that the science of Statics presents to us. A certain body is acted upon by one or many forces, each impelling it in a certain direction with a certain

strength. How shall I keep it at rest? How prevent any movement following the single or combined action of any number of forces?

25. Centre of Gravity.—A circle, poised on its centre point, will remain in equilibrium, because the weight of one-half exactly balances the weight of the other, just as any weight, placed in one scale will balance, and be balanced by, an equal weight placed in the other. So of a square: its middle is its balancing point, supposing the substance to be uniform.

Is there any similar point in figures of less regular shape, such as triangles, trapezoids, &c., or even in irregular figures? Experimentally, this may be answered very simply. Take any piece of wood, metal, &c., of any shape, and suspend it from any point in its circumference (using the term in its fullest sense, as the outline of *any* figure). Draw a line downwards directly from this point of suspension. Suspend it again from some other point, chosen at random, and again draw a line vertically downwards from this point. These two lines will be sure to cross, and the point where they cross is the CENTRE OF GRAVITY, or point on which the whole body will balance. This experimental discovery is quite independent of uniformity of substance.

It is certain that the part of the plate on one side of the vertical line, drawn from the point of suspension, will exactly balance (in point of weight) the part on the other side. For, if not, motion must necessarily follow; since the heavier part will weigh down the lighter, just as a heavier weight would overbalance a lighter one in a pair of scales. Therefore, if I suspend the plate of metal or wood by the hole (1), so that it is free to swing to and fro, it will finally come to rest, and a vertical line, *a b*, may be drawn, dividing the whole plate into two parts, A and B, equal in weight, and therefore balancing each other. If now I

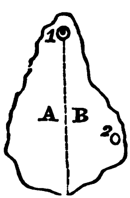


Fig. 26.

suspend it from any other point, say (2), it will again settle, so that the vertical line, *c d*, may be drawn, dividing the whole into two parts, C and D, of equal weight.

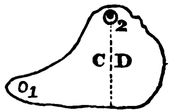


Fig. 27.

If I could find the exact point in *a b* round which the plate would balance, not only on each side of the line, but also at right angles to it, I should have the centre of gravity of the whole. It is evident that there is such a point in *a b*. It is also evident that there is also such a point in the line *c d*.

There cannot be two centres of gravity in the plate. There may be, and are, innumerable *lines* which divide the whole into equal halves, but there cannot be more than one point round which the whole can balance in every direction. Therefore, since this point is in *a b*, and also in *c d*, it must be at *o*, the point where these two lines cross each other. This point of intersection, *o*, is therefore the centre of gravity of the whole plate.

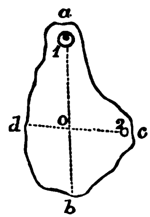


Fig. 28.

To find the centre of gravity of any body.—Suspend it successively by two points, making two lines vertically through the points of suspension. The point where these lines intersect is the required point.

26. Position of the Centre of Gravity.—In a circle the centre of gravity is evidently the centre: the same is true of a square. In a triangle it is in the line joining any angle with the middle point of the opposite side, and one-third of the distance from the base. If A B E be a triangle, and D the middle point of the base D E, then the centre of gravity of the whole triangle

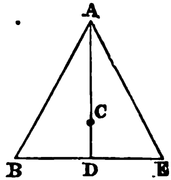


Fig. 29.

is at *c*, one-third of A D from D,—*i.e.*, A *c* twice *c* D.

If I have two triangles, then the centre of gravity of the two, taken as one body, is in the line joining the two centres of gravity. If A and B be two triangles, then the two, when fastened together, will balance about the point *c*. If the two triangles be of equal weight, then *c* will be half-way between them: if A be greater than B, *c* will be nearer to A; and if B be the greater, all will be nearer to B. In all cases

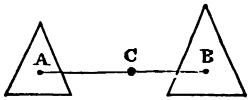


Fig. 30.

A *c* : *c* B : : B : A.

The same is true of two circles, a circle and a triangle, and generally of any two bodies, whatever be their sizes or shapes. It is also true of any number of bodies. If A B C and D be the centres of gravity of four bodies, the centres of gravity of the whole will be found by joining A and B, taking the centre of gravity of these two, then of that and the third, C; and finally the centre of gravity between A B C on one side and D on the other will be the centre of gravity of the whole group. Compare this with the method of finding the centre of parallel forces (page 30), when the two methods will be found to be identical. In fact, the centres of gravity may be considered as the points where gravitation acts on A B and C and D respectively. Since all these are drawn down to the earth, they may be considered as being acted upon by parallel forces. Compare it also with the method of finding the fulcrum of a lever from which several weights are suspended (page 54).

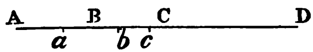


Fig. 31.

The centre of gravity of a parallelogram is at *o*, the

point of intersection of the two lines, A C, B D, joining the middle points of the sides opposite to each other.

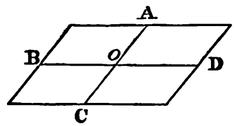


Fig. 32.

The centre of gravity of a triangle may be found by joining any angle with the middle point of the opposite side: where two such lines cross is the centre of gravity of the triangle.

The centre of gravity of a triangle may be also considered as the centre of gravity of the weight of the triangle collected in three equal parts, and placed at the angular points.

The centre of gravity of any rectilinial plane figure may be found by dividing it into triangles, and finding the centre of gravity of each. The centre of gravity of all these is that of the whole figure.

The centre of gravity of a triangular pyramid is found by taking the centre of gravity of the triangle forming the base, joining that with the apex, when the centre of gravity of the whole pyramid is in this line one-fourth from the base. It may be also considered as the centre of gravity of the whole weight collected in four equal weights placed at the four points of the pyramid.

The centre of gravity of a cone is found by joining the apex with the centre of gravity of the circle forming the base, and taking the point one-fourth from the base.

27. The Lever.—If I place two equal weights at the ends of a rigid bar of iron, which is capable of motion only about its middle point, they will evidently balance each other. Thus, if W and P be each 1 ℔ or 2 ℔s, or any other weight, and A C = B C, then the rod A B will be horizontal, and remain so.

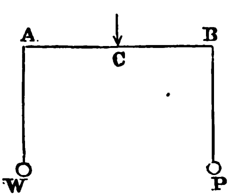


Fig. 33.

For to move it to any other position, as in fig. 34, : necessary to lower one weight and to raise the o and these two movem counterbalance and pre each other. W cannot go d unless P goes up; and P, b equal in weight to W,] downwards as much as W (and tends equally to raise

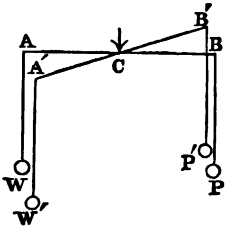


Fig. 34.

But if I make P hea than W, or W heavier tha I at once get motion. heavier weight falls, the lig rises.

If now I move the bar or lever, so that it rests n(the middle, but on some other point nearer to o] the ends, I find that my two equal weights no lo balance. I place C so A C is equal to half B C— at one-third of the w length of A B from A— fasten to A and B the e weights W and P. Imn ately P descends, and W r Why? It is customary tc that the moment of P a

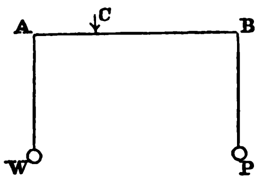


Fig. 35.

C is greater than the moment of W about C—*i.e.*, t

$$P \times BC \text{ is greater than } W \times AC.$$

Let W and P each = 3 lbs, A C = 2 feet, and =4 feet; then

$$P \times BC = 3 \times 4 = 12. \quad \text{And}$$
$$W \times AC = 3 \times 2 = \ \ 6.$$

$$\text{Therefore } P \times BC \text{ is greater than } W \times AC.$$

But the technical term "*moment*" is only a n for an *effect:* it does not explain the *cause* of motion.

Let the bar A B be moved from its position A B to A′ B′. For this to happen, W must be raised from W to W′, and P will fall from P to P′. But it is easy to show that the weight P has moved twice as far as the weight W. For A C A′ and B C B′ are similar triangles, and B C = A C × 2 ∴ B B′ = 2 A A′ and ∴ P P′ = 2 W W′. When A C = B C, then W W′ = P P′, and these two movements being equal, prevent each other, and neither takes place. But when P P′ is greater than W W′, motion results. But why? Any given weight falling any given distance generates a force that will raise an equal weight an equal distance, half that weight twice the distance, or double the weight half the distance. The fall of 10 lbs through 10 feet will raise 10 lbs through 10 feet, or 1 lb through 100 feet, or 100 lbs through 1 foot, or 20 lbs through 5 feet, or 5 lbs through 20 feet, &c. &c.

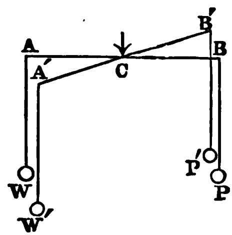


Fig. 36.

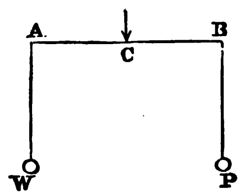


Fig. 37.

Now, if one arm be longer than the other, the extremity of that arm will move through a greater distance than the extremity of the other: consequently equal weights will not balance at these extremities, since the one at the shorter arm, moving through any distance, will be more than counterbalanced by the other, which will move through a greater distance.

How, then, can I balance two weights at the ends of

a lever of two unequal arms? By making the weights of such magnitudes as, moving through their respective distances, will balance each other. Thus, if one arm be half as long as the other, the weight at the end of that shorter arm must be correspondingly heavier than the other. Let A C be 1 foot, B C 2 feet, then a weight of 1 ℔ at P will balance a weight of 2 ℔s at W. For, in moving from the position A B to that of A′ B′, the bar will move W through a certain distance (say 4 inches), and P through twice that distance (say 8 inches). Now, 2 ℔s to be moved through 4 inches require precisely the same force as 1 ℔ does to be moved through 8 inches. This is why a small weight at the end of a long lever balances a larger weight at the end of a short lever. The extremity of the long arm moves through a greater distance than the extremity of the short arm, and consequently the weight attached to it does the same.

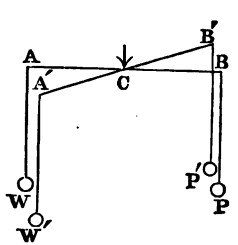


Fig. 38.

I can now explain, I hope clearly, why motion results when equal weights are attached to arms of unequal length. Let A C be 1 foot, B C 2 feet. Also, let W = 2 ℔s and P = 1 ℔. Here $P = \frac{1}{2} W$, and $A C = \frac{1}{2} B C$ and W balances P. I now add another 1 ℔ to P, so that W and P are each 2 ℔s. The 2 ℔s at W balance 1 ℔ at P, so that there is nothing to balance the second pound placed at P, and *this* (not the whole

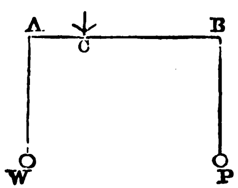
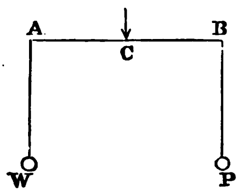


Fig. 39.

of the 2 lbs) is the weight that moves the bar. It is not that the whole of the weight at the end of the longer arm overcomes the resistance of the weight at the end of the shorter arm. But that *part* of the weight at the end of the longer arm is balanced by, and balances, the *whole* of the weight at the end of the shorter arm, and that the remainder has nothing to counteract it, and moves accordingly.

Therefore, when we have a lever of unequal arms, we have two questions to answer:—(1.) What weights will keep the lever in equilibrium? (2.) What will be the result of *equal* weights?

The answer to the first question is:—The weights must be inversely as the lengths of the arms. So that



$$W \times A\,C = P \times B\,C.$$

Fig. 40.

The answer to the second question is:—The motion will be the result due to the action of that portion of the equal weight at the end of the longer arm, which is in excess of what is required to balance the weight at the extremity of the shorter arm. That is—

$$P = W = \frac{W \times A\,C}{B\,C} + \frac{W \times A\,C}{B\,C}$$

Of these two portions, one balances W, and the other moves the lever. This particular equation is true only when one arm of the lever is twice the other.

28. Forces not at Right Angles.—I have a lever with equal arms, A C, A B, and two equal weights,

P and W; but one of these only, P, hangs vertically; the other, W, passes at an angle of 45° over a roller at N. How will this affect the balance? If it gives any advantage to either, to which? Is there any method of measuring the ratio of effective

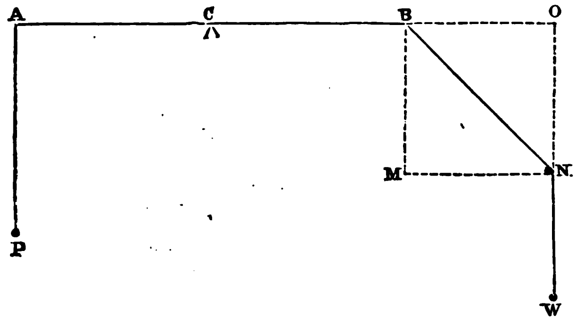


Fig. 41.

to non-effective force, when the weight acts at an angle less than 90°? Can I make the two arms of the lever remain in equilibrium by altering the weights?

Experiment shows that a pound weight hanging vertically has more force to move the lever than when it is suspended over a second point, as at N. The amount of the force that is effective in depressing the lever, B, may be found by drawing the triangle B M N, M N being parallel to A B, and B M at right angles to it. B M represents the force acting at right angles to the lever; M N represents the force acting in the line A B. This may be also shown by the line B O.

We may regard the three lines, B M, B N, B O, as the three lines of force—B M, B O, being the two forces

into which the actually applied force, B N, may be resolved.

If, then, the line B N represent 1 ℔ weight, the line B M will represent the effective force of that weight tending to depress the lever. The ratio of this to the whole weight will depend upon the angle M B N—the greater this angle the less will be B M. It also depends upon the angle N B O (the complement of M B N), the greater this angle the greater will be B M. The angle B N M is equal to the angle N B O, and the triangle B M N is equal in every respect to the triangle N O B, and is more convenient to discuss. It will be found that when B N M is 30°, then B M $= \frac{1}{2}$ B N, which represents that only half the force will act in depressing the lever. Therefore, it will require 2 ℔s at B acting at an angle of 30° to balance 1 ℔ at A acting at right angles.

From this it follows, any less weight than 2 ℔s—say $1\frac{3}{4}$ ℔ at B, would be weighed down by 1 ℔ at A. Is this a contradiction to our previous theorem, that, with levers of equal arms, equal weights balance each other? Is it also a contradiction to our previous theorem to find a less weight moving a greater one?

A little consideration will show that, so far from contradicting either of these, it confirms them both. It is true that 1 ℔ balances 2 ℔s, even with a lever having equal arms, but only when half only of the greater weight acts against the smaller. It is true that 1 ℔ will move any weight less than 2 ℔s, but only *through a correspondingly smaller space.* For, if we suppose the arm A to be depressed very slightly—say $\frac{1}{10}$ of an inch, and B consequently raised the same—it will be found that W does not move through so great a distance as P. If P be lowered $\frac{1}{10}$ of an inch, A will be lowered, and B will be raised, precisely the same distance; M B will be also increased $\frac{1}{10}$ of an inch, but B N will only be lengthened $\frac{1}{20}$, and W will consequently be raised only $\frac{1}{20}$ of an inch, just half the distance that P was

depressed. So that 1 lb. moving any given distance moves 2 lbs. through just half that distance. This is not at variance, but in strict agreement with, what we have before seen to be true in the case of unequal weights, balancing at the arms of a lever of unequal arms.

To sum up, therefore, we may say that any given weight, acting on a lever at an angle of 30, exercises

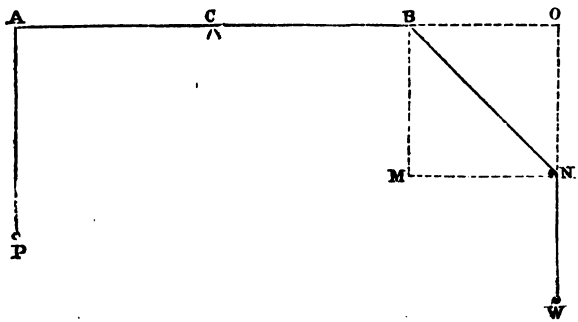


Fig. 12.

only half its total force in moving, or tending to move, the lever: consequently, to balance any given weight, acting vertically, twice that weight, acting at 30°, must be employed. Thus,

1 lb at 90°	will balance	2 lbs at 30°.
5 lbs at 90°	,,	10 lbs at 30°.
20 lbs at 90°	,,	40 lbs at 30°.

Will this hold good for any other angle than 30°? Try the experiment of an angle of 45°. The two questions to be answered are:—(1.) What ratio of 1 lb., acting at B, over N at 45°, will be effective in

depressing B? What weight, so acting at W, will balance 1 ℔. at P?

The two lines, B M, B O, are the respective forces, acting along the lever, and at right angles to it, and these are equal; for M B N and N B O are each 45°. At first, the idea will suggest itself that half the weight will act along B O and half along B M. But a moment's thought will show this to be wrong. If B N represent 1 ℔, B M and B O will each represent, not $\frac{1}{2}$ ℔, but the square root of $\frac{1}{2}$ ℔. For $B N^2 = B M^2 + B O^2$, and if $BN^2 = 1^2$, then $BN = 1$, $BM^2 = \frac{1}{2}$, and $BM = \sqrt{\frac{1}{2}}$: so also $B O^2 = \frac{1}{2}$, and $B O = \sqrt{\frac{1}{2}}$. Therefore, B M and B O each equal $\sqrt{\frac{1}{2}} \doteq \cdot 7$ nearly.

Therefore for every 1 ℔ acting on B at an angle of 45°, nearly $\frac{3}{4}$ ℔ will be effective in depressing the lever at B. So that nearly 1 $\frac{1}{2}$℔ must act at 45° to produce the effect of 1 ℔ at 90°.

We can now answer our two questions:—

(1.) What ratio will the effective weight have to the real weight, when the latter acts at 45°? The answer is, $\frac{7}{10}$ or ·7.

(2.) What weight must act at 45° to produce the same effect as 1 ℔ at 90°? The answer is 1·4 ℔, nearly $1\frac{1}{2}$ ℔.

As before, it will be found that if the lever be depressed at A, and therefore raised at B, that the distances through which the two unequal weights will be inversely as their magnitudes. Thus, 1 ℔ at A in moving 1 inch will move 1·4 ℔ at B ·7 inch. 10 ℔s at A in moving 1·4 inch will move 14 ℔s at B 1 inch. 1 ℔ at A in moving 1·4 inch will move 1·4 ℔ at B 1 inch.

If *both* weights act at a less angle than 90°, what will be the ratio between their effects? Let P act at an angle of 45° at A, and W at an angle of 30° at B. That is, the angle S A M = 45°, and the angle R B N 30°. For these we may take A M O and B N Q.

When P acts along A M, its effective weight along A O is found thus:—Effect in A O : total weight : : length

of A O : length of A M. When A M O = 45° we have, length of A O : length of A M : : $\sqrt{\frac{1}{2}}$: 1 $\therefore$ effect of 1 ℔

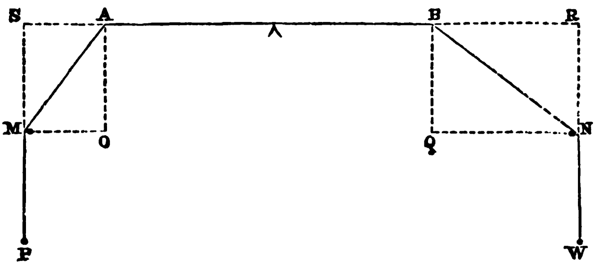


Fig. 43.

at P along A O $= \sqrt{\frac{1}{2}} = \sqrt{\cdot 5} = \cdot 7 = \frac{7}{10}$ ℔. In same way, 1 ℔ acting at W along B N has an effective force of ·5 or $\frac{1}{2}$.

To balance at A and B respectively, the weights at P and W must be such that $\frac{7}{10}$ of P = $\frac{5}{10}$ of W. Therefore W must be to P as 10 is to 7. That is, 7 W = 10 P $\therefore$ P = $\frac{7}{10}$ of W, and W = $\frac{10}{7}$ of P.

29. The Steelyard.—Since a large weight can be balanced by a small one, provided it act at a longer distance from the fulcrum, it is possible to have a weighing machine which shall require only one weight. This single weight will balance a small weight or a large one, by being placed nearer to, or farther from the fulcrum. Thus, if a bar, X Z, be suspended at a point *o*, so that *o* Z is four times as long as *o* X, and A B C are points dividing the longer arm into four parts, then a one-pound weight at A will equal and balance a one-pound weight at X; but at B it will balance

a two-pound weight at X; at C it would balance three pounds; and at Z four pounds. To balance a

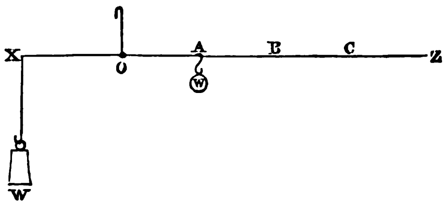


Fig. 44.

weight greater than one, but less than two pounds at X, the small weight would be put between A and B, and so on.

The weight of the bar X Z itself may be also made available as a measure of weight. If the fulcrum *o* were in the centre, the two arms would be of equal length; but *o* Z is much longer than *o* X; therefore the long arm being the heavier will tend to weigh down the shorter, and therefore, also, will tend to balance any weight, W, hanging at X. So that this weight W is balanced by two others, W and the weight of the whole bar X Z, which acts at its centre of gravity, which is between *o* and Z.

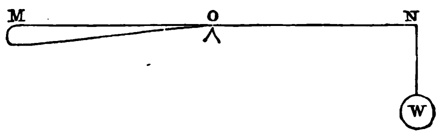


Fig. 45.

30. The Danish Steelyard.—The weight of the steelyard itself may be made so far available as to dispense with any other. Thus, if the arm *o* M be made much

heavier than the arm o N, a weight, W, will be required to keep the bar horizontal. But by this arrangement one weight only can be measured, that which when at N balances the extra weight of o M.

But if this inequality between o M and o N can be changed at pleasure, we can make it more or less until it compensates the effect of whatever we wish to know the weight of. Instead of moving the weight we move the fulcrum, until the inequality of the two arms just balances the weight to be measured.

In the common steelyard the fulcrum is fixed and the weight movable; in the Danish steelyard the fulcrum is movable and the weight is the effect of one arm of the lever over the weight of the other.

31. Equilibrium of a Body Resting on a Horizontal Plane.—We have seen that everything, whatever its shape or weight, has one point called its centre of gravity, round which the whole body balances. Also, we know that a cart, even heavily laden, may be drawn along a road one side of which is higher than the other, without falling over sideways. If the load be comparatively light, and so filling up a large space, the danger of its being overturned is greater than if it be heavy, and therefore low down in the body of the cart. Thus, a load of hay (fig. 46) is more easily overturned than a load of stone (fig. 47). In the one the centre of gravity is not supported, in the other it is. In either case, the cart will not be overturned so long as the line drawn vertically from the centre of gravity passes between the wheels, but will the moment it falls outside either of them.

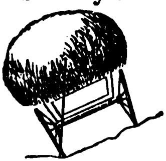


Fig. 46.

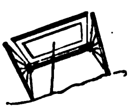


Fig. 47.

32. Stable and Unstable Equilibrium.—If, in the case of the load of hay or of stone, the centre of gravity be exactly over one of the wheels, the cart will not overturn, but the whole weight of the cart and load will balance

on that wheel, just as a plate might be balanced on the end of a stick, and the least force will overturn it. In the same way, any body which is supported on a small base is in danger of falling from the effect of a slight push, while, if the base be broad, the overturning effect of any given force is much reduced. Fig. 48 is an extreme case of *unstable* equilibrium, or danger of being overturned, while fig. 49 is an extreme case of the reverse, or stable equilibrium. The slightest push would overturn A, while B might be raised on one edge considerably, without any danger of being pushed over.

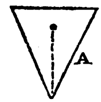


Fig. 48.

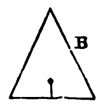


Fig. 49.

33. Laws of the Motion of Falling Bodies.—We have seen that any weight unsupported falls to the ground: in fact, it is the attraction of gravitation that gives it weight. Also, we have seen that this force acts

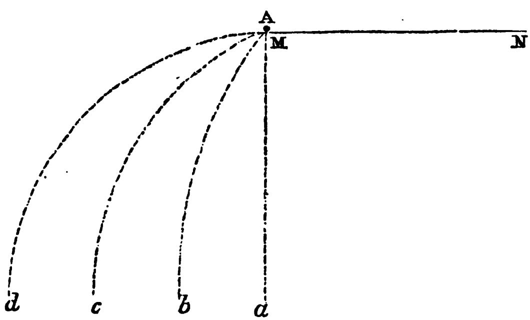


Fig. 50.

continually, so as to increase in effect upon a falling body. Thus, a stone falling from a tower falls more and more rapidly until it reaches the ground.

During the first second it falls through a space of 16 feet. This is a fact proved by actual experiment. If A be a ball resting on a table, M N, which is 16 feet above the ground, it will be found to take one second to reach the ground, whether it fall vertically to *a*, or be thrown forward to *b*, *c*, or *d*. That is, its downward motion is quite independent of any lateral motion, being the force of gravitation, while the other is the result of some specially applied force.

If the ball fell from the top of a lofty tower, it would pass through 16 feet in the first second, 64 feet in two seconds, and 144 feet in three, and 256 feet in four, and so on, increasing in velocity continually.

Since it falls with a continually increasing velocity, its rate is less at the beginning of any given second than at its end; also, its average velocity during any second is of course greater than its initial, and less than its final velocity for that time.

During	1 Second.	2 Seconds.	3 Seconds.	4 Seconds.
Its total fall is . .	16 feet.	64 feet.	144 feet.	256 feet.
Deducting space passed previous to last second,	0	16 ,,	64 ,,	144 ,,
Fall during last second,	16 feet.	48 feet.	80 feet.	112 feet.
Effect of gravitation during last second, .	16 ,,	16 ,,	16 ,,	16 ,,
Initial velocity of last second, . . .	0	32 feet.	64 feet.	96 feet.
Final velocity of last second, . . .	32 feet.	64 feet.	96 feet.	128 feet.

From this we see—

1. That the distance traversed in any second is that due to its initial velocity, with the addition of 16 feet from the continued gravitation. Thus,

	1st Second.	2nd Second.	3rd Second.	4th Second.
Initial velocity of .	0	32 feet.	64 feet.	96 feet.
Effect of gravitation, .	16 feet.	16 ,,	16 ,,	16 ,,
Total distance during	16 feet.	48 feet.	80 feet.	112 feet.

2. That the increase of space passed through is 32 feet for every second.

3. That the initial (and also the final) velocity of each second is 32 feet greater than that of the preceding second. I have used whole numbers for clearness. The correct numbers are 16·1 and 32·2.

If v be used to express the velocity,
g ,, ,, force of gravitation,
s ,, ,, the space passed through,
t ,, ,, the time, .

Then $$v = g\,t.$$

That is, the velocity equals the force of gravitation multiplied by the time. At the end of one second the velocity is 32 feet, which may be expressed by $32 = 32 \times 1 = 32$. At the end of two seconds the velocity is 64 feet, or 32×2. The effect of gravitation during one second is taken as a unit, expressed by g: and since a ball receives from gravitation a final velocity, in one second, of 32 feet, then $g = 32$. More correctly 32·2.

Also, $$s = \tfrac{1}{2}\,g\,t^2.$$

Or the space passed through in any given time is half the square of the time multiplied by the force of gravitation.

During	1 Second.	2 Seconds.	3 Seconds.	4 Seconds.
The total fall is, . .	16 feet.	64 feet.	144 feet.	256 feet.

$$\begin{aligned}
&\text{In one second } s = \tfrac{1}{2}\, g\, t^2 = \tfrac{1}{2}\, 32 \times 1^2 = 16;\\
&\text{In two seconds } s = \tfrac{1}{2}\, g\, t^2 = \tfrac{1}{2}\, 32 \times 2^2 = 64;\\
&\text{In three seconds } s = \tfrac{1}{2}\, g\, t^2 = \tfrac{1}{2}\, 32 \times 3^2 = 144;\\
&\text{In four seconds } s = \tfrac{1}{2}\, g\, t^2 = \tfrac{1}{2}\, 32 \times 4^2 = 256;
\end{aligned}$$

and so on.

Lastly, $v^2 = 2\, g\, s$. Or, $v = \sqrt{2\, g\, s}$.

That is, the velocity equals the square root of twice the product of gravitation into space.

At the end of	1 Second.	2 Seconds.	3 Seconds.	4 Seconds.
The velocity is . .	32 feet.	64 feet.	96 feet.	128 feet.

$$\begin{aligned}
&\text{At the end of 1 second} \quad v^2 = 2\, g\, s = 2 \times 32 \times 16 = 32^2 \therefore v = 32.\\
&\text{,, 2 ,,} \quad v^2 = 2\, g\, s = 2 \times 32 \times 64 = 64^2 \therefore v = 64.\\
&\text{,, 3 ,,} \quad v^2 = 2\, g\, s = 2 \times 32 \times 144 = 96^2 \therefore v = 96.\\
&\text{,, 4 ,,} \quad v^2 = 2\, g\, s = 2 \times 32 \times 256 = 128^2 \therefore v = 128.
\end{aligned}$$

So that g and t, or g and s, being given, we can find v; also g and t being given, we can find s.

34. Attwood's Machine.—Since any weight free to fall, falls through 16 feet in one second, 64 feet the next, it is difficult to experiment upon freely descending bodies, which pass through considerable spaces so quickly. But this difficulty can be overcome by artifice:—Two equal weights, A and B, suspended by a string over a roller, C, balance each other, however placed, just as a window remains open at any distance, its weight being

balanced by the weights behind the framework. If a small additional weight be placed on A or on B, it will descend more slowly than if falling freely, but in obedience to the same laws. Its velocity will increase regularly, and the three equations $v = g\,t$, $s = \frac{1}{2}\,g\,t^2$, $v^2 = 2\,g\,s$, may be experimentally verified, not as being absolutely true in this case (gravitation being partially counterbalanced by friction), but as being proportionately true. That is, if it fall 6 inches in one second, it will fall 24 inches in two seconds, and 54 inches in three. Here $g = 12$ inches instead of 16 feet, then $s = \frac{1}{2}\,g\,t^2 = 6$ inches for 1 second, 24 inches for 2, and 54 for 3, &c.

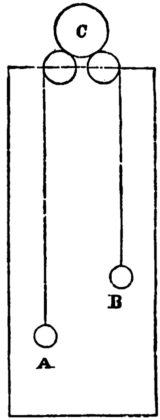


Fig. 51.

35. Morin's Machine. — The weight A falling freely, and having a small pencil projecting from it against the cylinder B, makes a straight mark, showing its vertical path. If any means could be devised of marking its position at the end of each second, its rate of increase could be clearly perceived. General Morin has done this by making the cylinder revolve horizontally with a regular velocity. If the weight were stationary and the cylinder revolving, the pencil would make a straight line horizontally. If the weight move, and the cylinder be stationary, a vertical line is made. If both move, the line made by

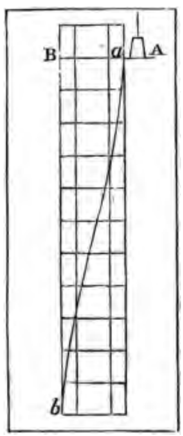


Fig. 52.

the pencil is between the two, and really becomes the parabolic curve, *a*, *b*. The cylinder is covered by a paper, which is divided, both horizontally and vertically, into equal spaces by parallel lines.

36. Uniform Motion in a Circle.—I tie a small weight to a string, and swing it round; it describes a circle as the resultant direction between two forces, one tending to keep it moving in a straight line, the other to draw it to my hand, which is at the centre of the circle. The stone, moving from D to E, would, if set free at E, continue in a straight line towards B. But the string constrains it. It therefore continues to describe the circle C D E, as the nearest possible approximation to a straight line.

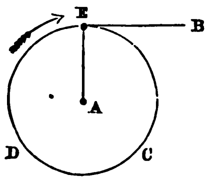


Fig. 53.

Since any weight so set swinging tends to get away from the circle, and to go off in a straight line outside it, the force so acting on it is called "centrifugal force" —*i.e.*, a force drawing it as far as possible from the centre of the circle. A carriage driven hastily round the corner of a street is in danger of being overturned, owing to its tendency to continue moving in a straight line. A horseman riding rapidly round a circus has to lean towards the centre, otherwise he would be thrown from the horse just as the stone would fly off in a straight line if the string were to break.

37. Small Oscillations of a Simple Pendulum.—I suspend a small weight by a string to a hook in the wall, and set it swinging. I find that whether it passes through a small space or a large one, it takes precisely the same time for every movement to and fro. If I draw it aside 3 inches, and then release it, I find it travels slowly; if 3 feet, it travels quickly; but it moves the whole distance, great or small, in the same time. I make the string about 39 inches long, and I find the weight

moves to and fro once every second. I move it quickly, it goes farther; I move it slowly, it goes a less distance, but takes always, whether quick or slow, exactly one second for every oscillation. I make the string longer, say 42 inches, it now takes a longer time to move to and fro; I shorten it, say to 30 inches, it now takes a less time.

Speaking generally, we may say, that the time required by a simple pendulum to oscillate depends upon its length, and upon its length only. Any given pendulum takes always the same time to move to and fro, whatever its rate of motion, and whatever the space through which it passes, always provided that the space be comparatively only a small arc of the circle, of which the point of suspension is the centre, and the length of the pendulum the radius.

38. Compound Pendulum.—I suspend six small weights from the same hook by strings of various lengths, so that I have six pendulums, no two alike as to length, and therefore no two taking the same time for each oscillation. Thus A, B, C, D, E, and F, are six simple pendulums, all oscillating according to the same law, but each independently of all the others. I pass a wire through all of them, so as to constrain them to move together (fig. 55). The shorter the string the more rapid the oscillations; the longer, the more slow. When free, A moves most quickly, F most slowly. When constrained, the times of oscillation are equal, since all move together; but F moves through the greatest space, and A through the least; therefore F moves the most rapidly, A the least so. So that the result of the restraint is to retard the motion of A, and to hasten that of F. Now B is retarded less than A, and C less than B. Also, E is

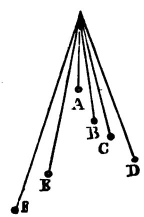


Fig. 54.

hastened less than F, and D less than E. Therefore there may reasonably be expected to be some point between A and F, which is neither hastened nor retarded, but moves as though it were quite free from all constraint, except that of the string of a simple pendulum. There is therefore one point of a compound pendulum (as a set of simple ones is called) which is still a simple pendulum, so far as its velocity is concerned.

Fig. 55.

Now, instead of a set of vibrating weights, constrained to move together by a wire, I suspend a rod of iron or of wood (upon two knife-edges, or in some other such way as to decrease friction) equal in length to F, the longest pendulum. It oscillates as the set of weights did, and one point (at the same distance from the point of suspension as before) moves as though free from all constraint but from simple suspension. This point is called the centre of oscillation, and will be found to be two-thirds of the length of the pendulum from the point of suspension.

A simple pendulum two-thirds the length of the compound pendulum will vibrate regularly with it, beat for beat: that is, the length of any simple pendulum is the same as the distance from the point of suspension of the *centre of oscillation* of a compound pendulum vibrating at the same rate. Thus a compound pendulum, 15 feet long, vibrates regularly with a simple pendulum 10 feet long, and its centre of oscillation is 10 feet from the point of suspension. If A B be a compound pendulum, its centre of oscillation is at C, so that A C is twice C B, and E D is a simple pendulum

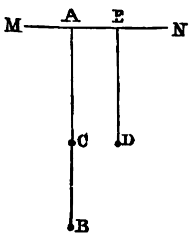


Fig. 56.

vibrating isochronically with it: E D being equal to A C.

In all these instances the pendulum has been supposed to be suspended from its extreme end. Suppose it to be suspended from some other point, what effect will this change have upon its rate of vibration? Suppose it to be suspended at *o*, then the part A *o* will move the contrary way to the part *o* B, falling when it rises and rising when it falls. The result of this is to decrease its rate of vibration.

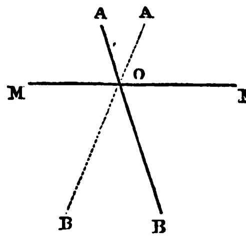


Fig. 57.

Let the compound pendulum A B (fig. 56) be inverted and suspended from C, its centre of oscillation. It will be thus the same length, below the point of suspension, as the simple pendulum, E D (fig. 56). It will be found that the two still vibrate together—that is, the compound pendulum vibrates with the simple pendulum two-thirds its length, whether it be suspended from A or from C, from one extremity or from its centre of oscillation. This is expressed by saying that the centre of suspension and the centre of oscillation of a compound pendulum are interchangeable—*i.e.*, if A be the point of suspension, C is the centre of oscillation; and conversely, if C be the point of suspension, A is the centre of oscillation.

By a simple pendulum is meant a weight suspended by a very fine flexible string; by a compound pendulum is meant any rigid bar suspended by any point. An ordinary clock pendulum would seem at first to be a simple pendulum; but it is a compound one. Being rigid, its various points are constrained to move together; and in point of substance, the thinnest wire commonly used is very thick compared with the ideal fine thread of a simple pendulum.

39. Determination of the force of Gravity.—What is the force that keeps a pendulum in oscillation? I move it with a very slight effort a little on one side, and it swings to and fro for a considerable time, especially if there be but little friction. Thus, I raise B to C, and release it. The force of gravitation pulls it down again to B, and it rises to C′ on the other side of B. If there were no resistance of air, and no friction at A, then B C′ would exactly equal B C, and in this case it would vibrate for an indefinite time. When it has returned to B, it is where it was at first; and if it remained there, the force I exerted would have produced no effect, which is impossible. Really, the same force which pulls it from B to C also raises it from B to C′, and keeps it moving to and fro, until the whole force has been dissipated in friction.

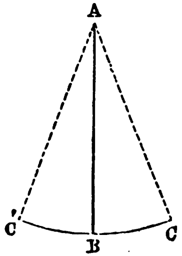


Fig. 58.

If I use more force in moving the pendulum, it goes farther from B, but the time of returning to B is the same. If the force of gravitation were increased, the fall from C to B and from C′ to B would be more rapid, that is, the pendulum would be pulled down with greater rapidity. So that the *space* through which the pendulum moves depends upon the force with which it is pulled aside; the *time* it shall take to move through the space depends upon the force of gravitation.

Therefore, any given pendulum will move to and fro in the same time, whatever the space (if comparatively small) through which it passes; but this time will vary if gravitation be increased or diminished. How can gravitation be increased or diminished? If I go farther from the centre of the earth by climbing a mountain, it is diminished; if I go nearer to it by descending a valley or a pit, it is increased. If I travel

from the equator, northward or southward, toward the poles, gravitation is increased; if I move from the poles toward the equator, it is diminished; for at the poles the surface of the earth is nearer to the centre than at the equator. Just as the whole weight of any body may be considered to act at its centre of gravity, so the whole attractive force of the earth may be considered to act at its centre.

Therefore the time in which a pendulum vibrates may be used to determine the force of gravity. By calculation (too complicated to be given here) it has been found that the expression—

$$3{\cdot}1416\sqrt{\frac{\text{Length of pendulum}}{\text{Force of gravity}}}$$

gives the time of vibration of a simple pendulum. Also by calculation, it has been found that the force of gravity at London is 32·1908—which means, that a freely falling body would at London have a velocity of 32·1980 feet at the end of the first second of its descent. So that a pendulum 3·2616 feet long ($3\frac{2616}{10000}$ ft.) will vibrate at London once every second, the force of gravity being (as we have seen) 32·1908. If now I take a pendulum of this length to any place where the force of gravity is less than at London, it will not, when pulled aside, be drawn down with so much force, and will therefore move more slowly, and so take longer to vibrate. If, on the contrary, I take it to any place where gravitation has more force, it will be pulled down with more power, and will therefore vibrate more rapidly than once a second.

So that at any place I can determine whether the force of gravitation be greater or less than at London, by setting in motion a simple pendulum of 3·2616 feet in length. Of course, there are very many precautions to be observed—in fact, the experiment is one of very great delicacy. But it is easy to see how a pendulum can be used to measure the height of a mountain above the sea;

or the depth of any great depression, such as a deep valley or a mine.

40.—Inequality of the Force of Gravity at Different Places.—The force of gravitation is the attraction of the centre of the earth (acting as the centre of gravity of the whole mass) for any body near it. The earth being not quite a globe, but a little flattened at the N. and S. poles, the centre and surface are nearer together at the poles than at the equator, where they are farthest apart. A weight weighing exactly a pound at the equator increases gradually in weight, as it is taken from E. to N. or from E. to S. Conversely, any given weight weighs less as it is carried from the N. and S. pole to the equator, until there it weighs least.

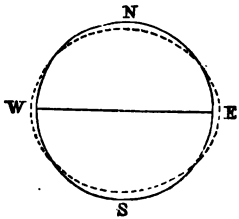


Fig. 59.

For the same reason, anything weighs less at the top of a mountain than at the bottom, and more at the bottom of a mine than at the top.

41.—Energy or Accumulated Work of a Moving Point.—I rest a pound weight on a sheet of paper supported on a frame: it remains at rest; but if I lift it and let it fall on the paper, it at once passes through. The force which enables it to do this it derives from me: the strength with which I raised it was really imparted to it, and enabled it to pass through the paper. It is as true that I break the paper as if I were to thrust my finger through it. Again, to drive a nail into wood, or to beat out the head of a rivet, I use a hammer. If I were to lay the hammer on the nail or on the rivet it would not have much effect; yet by raising it and bringing it down again quickly, I am able to produce an appreciable result. But the hammer itself does not become heavier, so that the force must be derived from my work. If a large stone were drawn up to the top of

a high tower, and then let fall, it would fall with all the force of the strength used to raise it. To set fountains at work water must be pumped up to some height, and let fall into the pipes of the fountain. In this case, also, the force used to pump up the water is really that which raises it a second time out of the fountain.

In every case force spent in one way is reproduced in another, and is really transferred from one place to another, just as a ball thrown against a window breaks it with the force transferred from the hand to the window; and generally force may be said to be absolutely indestructible. However it may seem to be dissipated or lost, it is still in existence.

42.—Law of Transmission of Pressure through a Fluid. —I fill a bag with fine sand, and tie the mouth tightly. I now open it just sufficiently to admit a thick piece of wood which I force in, while taking care that no sand comes out. Room can only be made for it by the sand being more closely packed together. Which part of it will be most compressed? Since the sand is fine, the particles will move freely about each other, and the pressure will be distributed equally over all parts of the bag. Had it been a solid, the pressure would have been in one direction only, as when a weight presses on a table.

This may be seen by placing side by side a table and a bag of fine sand, and placing a heavy weight on each. The pressure on the table will be downwards only, any light substance, such as a feather or paper lying on the table, would not be moved by the weight. But the pressure of the weight on the bag of sand would be seen to act on every part of it, since every part would swell out tightly, and a feather on any part would be moved.

The sand is really a fluid, practically, as the particles move so freely over each other; and this may be shown by making a hole in any part of the bag, when the sand will at once be forced out. If any number of holes be made at the same time, and of the same size, the sand will come out of all with equal force. Or if a vessel,

having a lid with several holes in it, be filled with water, have a piston forced into it, the water will be forced out of all the holes in the lid, just as freely as from holes of the same size made in any other part of the vessel.

Generally, we may say that pressure upon a fluid is transmitted equally in all directions, and at right angles to the surface upon which it acts.

43. Pressure of a Fluid against a Plane Area. Centre of Pressure.—I have assumed the sand to be in a bag; let it be in a square box instead. Will there be any difference between the pressure of the different parts of the side? Also, will there be (as in the case of parallel forces, p. 30) any point which might be called the centre of force or of pressure? The pressure will be equal on every point, excepting that there will be the weight of the upper part upon the lower, so that the lowest stratum will be subject to two forces—its own weight and the pressure from above. This will also be true of every stratum (assuming the whole contents to be divided into an infinite number of slices or strata); therefore the pressure on each side will increase regularly from above downwards. Thus, suppose the whole contents to be divided into parallel strata, *a*, *b*, *c*, &c., then any pressure brought to bear on it is diffused equally throughout, and *a*, *b*, *c*, *d*, *e*, and *f* all press against the side with equal forces. But, besides this, *b* has to bear the weight of *a*, and *c* the weight at *a* and *b*, *d* the weight at *a*, *b*, and *c*, and so on; so that each stratum is pressed with greater force than any above it, and with less than any below it. Therefore, the pressure of *a* against the side is the least, and of *f* is the greatest; and, therefore, the centre of pressure (corresponding with the centre of force) is below the centre of the side,

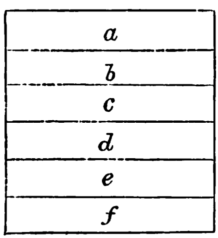


Fig. 60.

but also in a straight line below it, since the pressure is without any horizontal variation.

44. Pressure of a Fluid on a Body wholly or partly Immersed.—I throw a piece of iron into a basin of water, and it at once sinks to the bottom; I throw in a piece of cork, which floats at the top, but partly immersed. The iron sinks because it is heavier than water; the cork floats because it is lighter, but it is partly immersed, because it has some weight. The water being a fluid, its particles have little or no cohesion; so that anything placed on water is supported only by the particles immediately beneath it. If it be heavier than water, it is drawn down through it; for being heavier means being drawn down with more force. But its downward progress is resisted by the water in its way, which has to be displaced. Thus, *a* (fig. 62) cannot sink into the space *c* without the level of the water being raised from *o* to *o′*. This is resisted by the same force, gravitation, that pulls down *a*. That is, the weight of the water acts against the weight of the iron; but the latter being the greater, prevails, and the iron descends into the space *c*. But it continues to descend into the spaces *d*, *e*, and *f* successively, until it rests on the bottom of the vessel containing the water, and the level of the water does not rise above *o′*. But it has to be driven out of the space into which the iron passes; and this can only be done by raising the water from below the iron to above it—from the space into which it passes to the space which it leaves. So that in every case the weight of the water partly counterbalances the weight of the iron, and, in consequence, the iron is drawn through water with less force than through air. This is usually expressed by

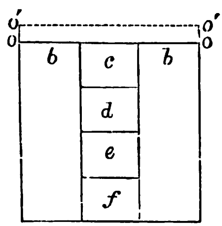


Fig. 61.

saying that iron weighs less in water than in air. The same is true of all substances.

But it is not true that iron weighs less in water than in air: it passes through water more slowly than through air, because the difference of the weights of iron and water is less than the difference of the weights of iron and air; and it is this difference of the weights only that is effective in moving the iron. The iron weighs the same but its weight is partly counterbalanced.

The cork, however, also sinks somewhat into the water, although not so heavy. Why? If the water be heavier than the cork, why does the latter descend at all, since it cannot do this without moving some of the water? If I put a large weight into one scale, and then a small weight into the other, the larger weight will be moved some distance by the smaller, though not far enough to bring the scales level. Just in the same way the lighter cork moves the heavier water in some degree, but not so much as if it were iron or lead. For the cork, a, is pulled down by gravitation, but cannot descend without displacing the water immediately below it. In sinking a little way into the water, it displaces less than its own weight of water, the level of the latter rising from o to o', and again from o' to o''. This stratum of water between o and o'' weighs less than the cork, and is therefore displaced by it; but the next stratum is kept in its place, not only by its own weight, but also by the weight of o, o'', now raised above it, and resting on it. So soon as the weight of these two—the stratum immediately below the cork and that of the water already displaced—equals the weight of the cork, the latter will sink no farther. But the weight of the stratum of water immediately below the cork may

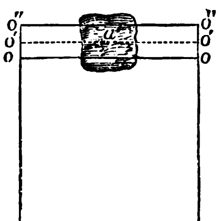


Fig. 62.

really be disregarded, so that the cork will sink until it has displaced its own weight of water, and then come to rest.

45. Specific Gravity of a Solid or Liquid.—Iron is heavier than water, cork is lighter—*i.e.*, gravitation has more power over a given volume of iron than it has over an equal volume of cork; therefore iron is said to have a greater specific gravity than cork; in other words, the specific gravity of iron is greater than the specific gravity of cork.

It would be easy to arrange the names of all substances in order—the heaviest at one end of the list and the lightest at the other. But this would not give more than a general idea of their respective weights. Just as in the case of weight, of time, and of space, we want a *unit of specific gravity*, in terms of which to express the specific gravities of other substances. The weight of water is usually taken as this unit. If a body be heavier than water, it is said to have a greater specific gravity; and if it be lighter, to have a less specific gravity. In the case of a gas, the weight of air is taken as unity.

46. Determination of Specific Gravity in Simple Cases.—The weight of water being the standard of comparison, it is necessary, in order to ascertain the specific gravity of any given substance, to know the exact weight of an equal volume of water with which to compare it. To do this several methods are available—(1.) To weigh the substance first in air (or in vacuo), and then in water, by the hydrostatic balance; (2.) To use the specific gravity flask; (3.) To use one or other of the many hydrometers that have been invented.

47. The Specific Gravity or Hydrostatic Balance.—It is required to know the specific gravity of S, a body insoluble in water, such as iron or platinum. It is weighed in the ordinary way by being placed at one end of the beam, A B, of a pair of scales, and its gravitation balanced by weights placed in the other scale. The beam is then lowered until the body S

descends into the water in the vessel, W (or the water may be raised instead of the beam being lowered). The weights at C now more than counterbalance S, for the water also more or less counterbalances its weight. The difference is really, as we have already seen, the weight of a volume of water equal to that of S. Thus, 22 ounces of platinum would weigh but 21 ounces in water; therefore 1 ounce is the weight of an equal volume of water. Then,

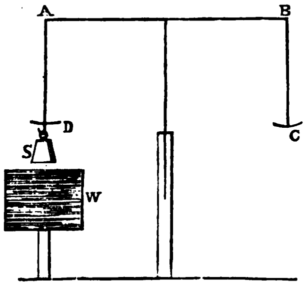


Fig. 63.

oz. oz.
As 1 : 22 : : specific gravity of water : specific gravity of S;
or, As 1 : 22 : : 1 : specific gravity of S;

for the specific gravity of water is always 1, being the standard.

Therefore $\frac{22}{1} = 22 =$ specific gravity of S;

or the specific gravity of a solid, insoluble in water, may be found by dividing its weight in air by what it weighs less in water. If W be its weight in air, and W′ its weight in water, then $W - W'$ is the expression for its loss of weight in water, and therefore for the weight of an equal volume of water, and $\frac{W}{W - W'}$, = its specific gravity. But the expression is derived from the proportion above stated. Of course, the water should be distilled, and the temperature of 4° C. is usually kept during the weighing.

But the substance, S, may be soluble in water, as sugar, or salt, or sodium. The method then is to use, instead of water, some liquid in which S is not soluble.

Thus the sodium may be weighed in naphtha. In water it would float, but in naphtha it sinks; for it is lighter than the one, but heavier than the other. Being weighed in it, we find its specific gravity compared with naphtha; and then, knowing the specific gravity of naphtha as compared with water, we can estimate that of sodium. Thus, 9 ounces of sodium would displace about 8 ounces of naphtha, and so weigh only about 1 ounce. Then $\frac{9}{9-1} = \frac{9}{8} = 1{\cdot}125$, which is, roughly speaking, the specific gravity of sodium as compared with naphtha. But a volume of water equal to the naphtha would weigh about 10 ounces; so that it is really $\frac{9}{10}$, not $\frac{9}{8}$, which gives the specific gravity of sodium = about ·9, or $\frac{9}{10}$ that of water.

Had the sodium not been soluble in water (or rather combustible, for sodium placed in water immediately burns and combines with the oxygen of the water), it might still have been weighed in it, being prevented from floating by being fastened to the scale.

But the body to be weighed may itself be a liquid. In this case one method is to weigh some solid body in air, in distilled water, and in the given liquid. This process gives the respective weights of equal volumes of water and of the liquid to be measured: the ratio of these two is its specific gravity.

48. Nicholson's Hydrometer.—To save the trouble of having a beam and pair of scales, a hollow cylinder of tin or zinc, *a*, having a weight, *b*, depending from it, serving to keep it vertical, is made so as to be a little lighter than water. The upper part, *c*, is therefore above the surface, and is kept at some fixed distance from it by putting small weights on *c*. This keeps the point *t* level with the surface. The body to be weighed is placed on *c*, and the point *t* then sinks, but may be raised again by taking from *c* weights

equal to *s*. The weight of *s* in air is thus known; it is then moved from *c* to *b*, so as to be under water. The point *t* now rises above the surface, since *s* does not weigh so much under water as above it. The weights that have to be now placed on *c*, to bring *t* back to its proper level, show what *s* has lost by being placed in water, and so shows what an equal volume of water weighs. The ratio of the weight of *s* in air to the weight of an equal volume of water is its specific gravity.

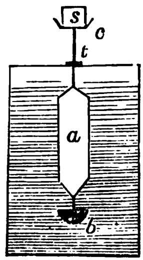


Fig. 64.

49. Hydrometers of Variable Immersion.—In all the instruments for ascertaining specific gravity hitherto described, the same principle has obtained of noticing what weight has been required to restore the hydrometer to its normal position. But evidently the same result might be attained by noticing how far the instrument is raised or depressed by an increase of its own weight, or of density in the liquid in which it is immersed. Thus, a hollow cylindrical body, *a*, may have a stem, *b*, so graduated as to show the relative specific gravity of a liquid in which it is placed. If the liquid be heavier than water, the hydrometer will not sink so deeply, since a less quantity of the liquid will balance its weight; if it be lighter, the instrument will sink more deeply than in water. The stem may be graduated (by actual experiment, after the manner of a thermometer) so as to show by simple inspection the specific gravity of the liquid.

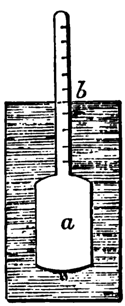


Fig. 65.

50.—The Specific Gravity Bottle.—If the substance to be weighed be a powder, none of the methods already mentioned are exactly applicable. One way would be to fill a bottle of known capacity with the powder and to compare the weight with that of the same bottle full of water. But it is difficult to say when a bottle is fairly full of a powder, which is so much more capable of compression than a liquid. The plan usually adopted, therefore, is to weigh both some given quantity of the powder and a bottle filled with distilled water. These are put together in one scale, and the total weight noted. Then the powder is put into the bottle, some of the water being displaced by it. The amount so displaced of course equals the volume of the powder, and the bottle is now weighed again. The weight is now less by the amount of water displaced, and thus the weight of a volume of water equal to the powder is known. The ratio of the weight of the powder to the weight of an equal volume of water is thus known, and this ratio is the specific gravity of the powder.

51.—Conditions of Equilibrium of a Floating Body.—A piece of cork will float in the roughest sea, so will a corked bottle, because nothing can make them heavier than water. But if the cork come out of the bottle, and it fill with water, it will probably sink at once. So a ship, if it fill with water, probably becomes heavier than the same volume of water, and therefore sinks. It may do this by a leak being made, or by being so far overbalanced that the water enters from above.

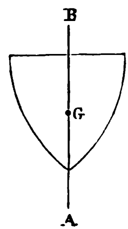


Fig. 66.

But, as in the case of a cart on a road (page 54), the danger of this depends upon the position of the centre of gravity of the whole vessel. Thus, if the line A represent the resultant of the upward pressure of the water (that is, its resistance to being moved by the weight of the floating body), and G be the centre of gravity, then

the vessel is in complete equilibrium when G is in the line A, as in fig. 66; if the vessel be disturbed, as in fig. 67, G acts so as to restore it to its normal position; but if it be as in fig. 68, with the centre of gravity high up, its action is rather to help to overturn the vessel.

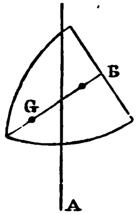


Fig. 67.

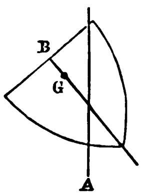


Fig. 68.

52.—The Meta-centre.—The line B, in which is the centre of gravity, G, of the floating body, is the axis of the whole body, which balances about it. The line A is the resultant of the force of the water acting upwards upon the body. When these two coincide, the floating body is in its safest position; if they do not coincide, as in figs. 67 and 68, the point where they intersect is called the meta-centre. If this point be above the centre of gravity as in fig. 67, it is evident that the weight of the vessel acts towards restoring it to its normal vertical position, but if it be below G, as in fig. 68, then it evidently acts towards overturning the vessel.

So that we may say that a floating body is in a position of stable equilibrium only when its meta-centre is above its centre of gravity.

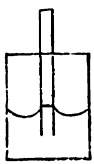


Fig. 69.

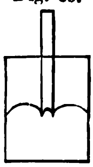


Fig. 70.

53.—Capillary Elevation and Depression.—If I dip a glass rod into water, some of the liquid will adhere to it when I take it out again; but the same rod dipped in mercury comes out perfectly free. Again, I dip a glass tube first in water, then in mercury: in the one case the liquid rises in and around the tube, in the other it falls. The water is attracted by the glass, and clings to it as shown in fig. 69, while the mercury is repelled by it as shown in fig. 70. Secondly, I fold a piece of blotting paper, as shown in

fig. 71, and dip it in water. The water rises in the way shown, higher at the fold than at the open part. Also, if I dip in water two tubes, one larger than the other, it will be found that the water rises higher in the one of small diameter than in the other.

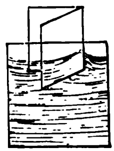


Fig. 71.

Generally, if a tube be dipped in a liquid which will wet it (*i.e.*, a liquid for which it has an attraction), the surface will be raised both inside the tube and outside, as shown in fig. 69; if it be dipped in a liquid which will not wet it (*i.e.*, one for which it has no attraction), the surface will be depressed both inside and outside, as in fig. 70.

54.—The Law of Diameters.—Also, the height to which the liquid rises or is depressed varies with the diameter of the tube, the smaller the bore the greater being the rise or the depression, as in fig. 72. Generally, the extent of this varies inversely with the diameter of the tube: if one tube be half the diameter of another, the rise in the smaller will be double the rise in the larger, and *vice versa*. If one tube be three times as wide as another, the rise in it will be but one-third of the other.

Fig. 72.

55.—Proof that Air is a Heavy Elastic Fluid.— That it is a fluid, is shown by the readiness with which it enters by any opening, however small, by the fact that it fills any vessel, whatever be its shape, and that it mixes readily with any other gas. That it is heavy, may be shown in many ways. A bottle filled with air weighs more than when it was perfectly empty. The receiver, *a*, of an air-pump is inseparably fastened to the table, *b*, when the air is pumped out from within it, by the pressure of the air upon the outside. Water may be transferred from one bottle to another, from which the air has been pumped, by the weight of air on the surface of

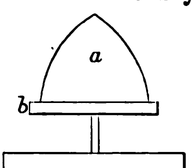


Fig. 73.

the water. A piece of thin glass, from one side of which the air has been removed, will be broken by the pressure of air on the other. And, generally, the fact that air is heavy (not in the sense of having great weight, but as having weight) may be shown by its pressure on any substance from the other side of which the air has been removed.

That air is elastic may be shown by the way in which a bottle partially filled with water may be emptied by the expansion of the air above it: the air, *a*, expands when the air is exhausted from the receiver under which the bottle is placed, and the water, *b*, is forced through the tube, *c*, until the surface of the water is below the end of the tube which passes through the cork. The mere weight of the air would not be enough to do this, since the weight of the water in the tube would more than counterbalance this. The air expands—that is, occupies more space than at first—as soon as the pressure through the tube from the outside air is removed, showing that it was kept in the space, *a*, by the pressure of the water, *b*, and the external air.

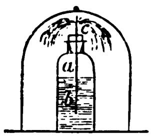


Fig. 74.

Again, pour a little mercury into the bottom of a bent tube (fig. 75), open at *a* but closed at *b*, so that the air in the short arm at *b* is enclosed and separated from that in the long arm, *g*. Supposing the long arm, *a*, to be at least 5 feet in length, pour mercury in at *a* until it reaches *m*, 30 inches from the bend. The air at *b* will be compressed into *b o*, just half the space it had previously occupied. This shows that air is compressible; pour the mercury out again by degrees, and the air will, by equal degrees, re-expand to its first dimensions—thus showing that it is elastic. Pour in 60 inches of mercury, up to *n*, and now the air at *b* will be compressed

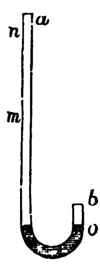


Fig. 75.

into one-third its original space, but will expand again as the mercury is removed.

Thirty inches of mercury is about equal in pressure to the atmosphere; so that the small column of air at *b* is first subjected to the ordinary pressure of the atmosphere, then to this pressure and that of 30 inches of mercury, and finally to this pressure and that of 60 inches of mercury—that is, the pressure is first doubled and then trebled. The air is compressed into half its previous space when the pressure is doubled, and into one-third when the pressure is trebled. Generally the volume of air varies *inversely* with the pressure to which it is subjected.

56. Torricelli's Experiment.—Centuries ago it was known that water could not be raised by an ordinary pump beyond some 32 feet; but this was not supposed to be connected in any way with the pressure of the air. Torricelli is said to have first shown that the weight of the atmosphere on the water in a vessel, *m*, kept a column of water in a tube, *a*, closed only at the upper end, *b*, of the height of some 32 feet. It occurred to him that if this was really the effect of the air's pressure, it ought also to balance a corresponding weight of mercury—*i.e.*, a column some 30 inches in height. He filled a tube, about a yard long, with mercury, and inverting it in mercury, found the column to fall until it reached a height of 29 inches. The upper (closed) end of the tube was a vacuum, except so far as the vapour of mercury filled it.

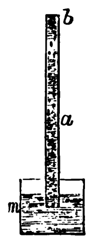


Fig. 76.

57. The Cistern Barometer.—The discovery that a column of any liquid could be kept in a tube closed at the upper end by the pressure of the external air at once suggested that the height of such a column would vary with this pressure, and that by this means we might have a visible measure of these invisible changes. A

column of mercury in a closed tube, standing in an open vessel of mercury, is therefore a barometer. It is necessary to keep both vessel and tube steady, and that the latter be perfectly vertical; since otherwise the whole weight of the mercury would not act against the air, but would be partly supported by the side of the tube.

To avoid the difficulty of keeping the mercury safe and pure in an open vessel, such as *m*, having a straight tube, *a*, closed at *b* (fig. 76), a bent tube, *a*, *b*, *c* (fig. 77) is used; the weight of the air being shown by the height of the column, *a b*, above the level, *b c*. The tube is closed at the upper end (the part above *a* being void of air), but is open at *c*, where the pressure of the air on one side of the lower portion of mercury, below *b c*, balances the weight of the column of mercury, *a b*, resting on the other side. The tube itself may be graduated, or it may be fastened to a wooden frame which is properly graduated. The usual marks are in inches in England and centimetres in France.

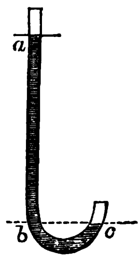


Fig. 77.

58. **The Aneroid Barometer.**—If I sit in a well stuffed easy-chair, the cushion is compressed by my weight, but expands again as soon as it is removed. So, in the same way, a metal box, quite empty even of air, would have its lid (especially if of thin metal) pressed down by the weight of the air, and the pressure would vary with the weight of the air. It must be borne in mind that the pressure of the air is very considerable, because of the enormous quantity of it. On every square inch the pressure is about 15 lbs; so that on the lid of a box three inches in diameter the pressure would be nearly a hundredweight. Such a box is the chief part of the aneroid barometer; the lid, aided by a spring, moves up and down as the pressure upon it varies, and these movements, small in themselves, are magnified in effect

by a set of levers, until they suffice to move an index, something like the hand of a watch, on a scale marked from about 28 inches to about 31.

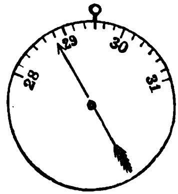


Fig. 78.

The aneroid barometer differs, however, from the mercury in one very important particular. Mercury is always the same; but no two metallic box-lids can be made so exactly alike as to move up and down with exact similitude. So that each one has to be marked for itself by actual experiment, and also frequently corrected, as very many things may happen to alter its elasticity. But, on the other hand, it is very much more portable than the mercury barometer, especially for mountain measurement.

59. Boyle's or Marriotte's Law.—If I pour a little mercury into a U tube, it will sink to the bottom, and, being pressed equally on each side by the air, it will stand at the same level in the two sides. If now I fasten the end of one side, I have a column of air cut off from all special external pressure, and occupying exactly the space it would fill if it formed part of the general atmosphere. But I can subject it to an increased pressure by pouring more mercury into the other tube; for, whatever be the amount of mercury present, it will try to stand at the same level in both arms of the tube; and, since the air cannot get out at the closed end, it will have to bear the pressure of the mercury trying to rise, and will thereby be compressed.

The air occupied originally the space it naturally filled under ordinary conditions—*i.e.*, subjected to the pressure of the general mass of the atmosphere. But we have seen that this is equal to the pressure of a column of water about 30 feet, or a column of mercury about 30 inches in height. But if these columns be of the same diameter their weights will be equal; and it is found by actual measurement that the weight of

either (when an inch in thickness) is about 15 lbs. Therefore the pressure of the general atmosphere upon every square inch of any surface exposed to it is about 15 lbs.

If now I pour an additional 30 inches of mercury into the open arm of the tube, I subject the air enclosed in the other to double the original pressure, since the column of mercury has exactly the effect of another atmosphere, so far as mere weight is concerned. And I find that, by thus doubling the pressure, I have also halved the volume; for, whatever space it originally occupied, it is now compressed into half of it. Thus, if it at first filled 12 inches, it now fills only 6, being halved in volume but doubled in density.

If I add another 30 inches of mercury — *i.e.*, 60 inches in all—and thus subject the enclosed air to a pressure equal to that of three atmospheres, I find I have compressed it to one-third of its original volume, or 4 inches. If my tube be long enough to hold it, I can, by the addition of another 30 inches of mercury, decrease the volume to one-fourth—*i.e.*, to three inches.

The original volume being	12 in.,	under pressure of	1	atmosphere,
30 in. of mercury reduces it to	6 ,,	,,	2	,,
60 ,, ,,	4 ,,	,,	3	,,
90 ,, ,,	3 ,,	,,	4	,,
150 ,, ,,	2 ,,	,,	6	,,

Considering the weight of 30 inches of mercury as equivalent to the weight of the atmosphere (upon an equal base), we may speak of the above as so many atmospheres. Therefore, if the pressure of one atmosphere confines the air in the tube to 12 inches, the pressure of two reduces it to 6 inches, and so on. Notice, that though I add an equal amount of pressure each time, the compression becomes less and less each time.

Thus, 1 atmosphere gives		12 inches.		
2 ,,	reduces it from	12	to 6,	or 6 inches less.
3 ,,	,,	6	to 4,	or 2 ,,
4 ,,	,,	4	to 3,	or 1 ,,
5 ,,	,,	3	to 2·4,	or ·6 ,,
6 ,,	,,	2·4	to 2,	or ·4 ,,

So that each additional weight does less and less work in the way of compression—*i.e.*, the more compressed the air is, the less additional compression does any given pressure produce; and this is true, not only of air, but of any gas whatever.

Or this law may be put another way. A given volume of air, or any gas, being 12 inches high, under ordinary pressure, the additional pressure of 30 inches of mercury will reduce it to 6 inches, or one-half. But to halve this volume—*i.e.*, to reduce it to 3 inches—not 30, but 60 inches more mercury is requisite. This, however, is not a departure, but the observance of the law; for if with a pressure of one atmosphere we have 12 inches, and with a pressure equal to two atmospheres we have half that volume, or 6 inches, so, also, if a pressure of two atmospheres gives a volume of 6 inches, then double that pressure, or a pressure equal to four atmospheres, is required to reduce this volume to one-half, or to three inches.

The original volume being	12 inches,
30 inches of mercury reduces it to	6 ,,
60 ,, ,,	3 ,,
120 ,, ,,	$1\frac{1}{2}$,,
240 ,, ,,	$\frac{3}{4}$,,

and so on—any given volume being reduced to one-half by the addition of a pressure equal to that then upon it, or may be doubled by the pressure being reduced one-half.

This principle is known as "Boyle's Law," or "Marriotte's Law," from the fact that Boyle and Marriotte each discovered and propounded it; and

the phrase, "Boyle's Law" or "Marriotte's Law," is understood to express all that has been now described. It may be also expressed in symbols.

Let P stand for any pressure; then twice that will be 2 P; three times, 3 P, &c.; also one-half the pressure will be $\frac{P}{2}$; one-third, $\frac{P}{3}$, &c.

Then, let V be any given volume, and we may express twice that volume by 2 V; one-half of it by $\frac{V}{2}$, &c.

60. Unequal Compressibility of Different Gases.—The law just described, that "the volume of a gas is inversely as its pressure," is not absolutely true for all gases under all circumstances. Thus, hydrogen and nitrogen are, under great pressure, not so compressible as air; while carbonic acid, ammonia, hydrogen-sulphide, and some others are more compressible. This might be expected from the fact that the less compressible gases are elements, while the more compressible are compounds. Also, the condition of a gas affects its compressibility: it is more compressible when near its point of liquefaction than when very rare. This, also, is what might be expected, seeing that the nearer a gas approximates to the liquid state, the greater is the effective action of gravitation between the particles. Lastly, the higher the temperature the more exactly is the law true, and the lower the temperature the greater the variation from it. This again might have been expected.

61. Limitation of Boyle's Law.—Therefore, we may say that Boyle's Law (as it is usually called in England) is true for all gases under small pressures, and in all common experiments; but that it does not hold for extreme conditions, and that the variations are greatest in the case of compound gases, when near the liquid condition; and least in the case of simple gases at high temperatures.

62. The Compressed Air Manometer.—Since air is compressed according to a regular law, its degree of com-

pression may be used as a measure of the pressure to which it is subjected. Thus, if it be desired to ascertain the pressure of any given body of gas confined in a vessel, it may be made to act upon a column of air, the compression of which will show the pressure exerted upon it by the gas.

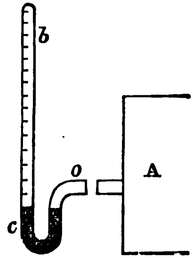


Fig. 79.

If the large vessel, A, be filled with a gas, the pressure of which it is desired to know, a tube, *b*, having its lower part filled with mercury, may be connected with it at *o*. The mercury in *c* is at the same level in both arms of the tube, being acted upon by the ordinary pressure of air on each side. If the gas in A have a greater pressure than the air, the mercury will be forced up in the tube *b*; if it have a less pressure, the mercury will be forced down by the air in *b*. The degree of pressure will be shown by the extent to which the mercury stands in *b* above the level in the shorter tube. The tube *b* is graduated so as to show the degree of compression to which the air is subjected, the pressure of the atmosphere (15 lbs to the square inch) being the usual unit, so that it is said to be under a pressure of two, three, or four atmospheres, &c.

63. The Suction-pump.—We have seen that water, mercury, or, indeed, any liquid, will flow upwards into a vacuum tube to which it has access (page 79). Thus, an empty tube, *a* (by empty, I mean void even of air), is placed in the vessel *m*, with its lower end open and the upper end closed. Any liquid that may be in the vessel *m* will be immediately forced, by the pressure of air on its surface, into the tube, until either the tube be full, or the weight of the liquid in it balance the pressure of air.

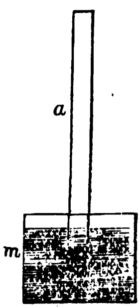


Fig. 80.

But, suppose the tube *a* to be closed above, not by the end of the tube, but by a piston, *o*, moving air-tight in the tube, and having an opening, closed by a valve, in its centre; then, if, when the tube be filled by the liquid, the piston be forced down by the handle *c*, the water or mercury will pass through the valve into the upper part of the tube. When the piston is again raised, the weight of the water above will close the valve at *o*, and the water will be drawn out of the tube at any opening, as at *s*. But, also, as the piston rises, more water will be driven into the tube by the pressure of the air, and this will in turn pass through the valve *o*, as the piston descends, and will be raised with it to *s*.

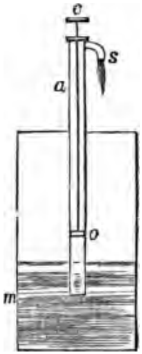


Fig. 81.

64. Hydraulic Press.—Water being almost incompressible, any vessel once full of water cannot be made to contain more. If we force more into it, the vessel must either be enlarged or broken. If there be any portion of it movable, that will be moved when more water be forced in. Thus, the large tank or cylinder, C, is filled with water, and is closed by a heavy lid, T, having a large piston, P, which reaches nearly to the bottom of C. When more water is driven in through the pipe, *o*, the piston, P, is forced up (*i.e.*, really forced out of the vessel, C, to make room for the incoming water), and with it the top, or table, T. But the table and piston are heavy, so that considerable force has to be applied to force the water into C. The piston, *p*, working water-tight in the cylinder, *t*, is used for this purpose. When it is raised, water flows in through the valve, *v*, which falls when *p* is forced down: *t* being full of water, *p* can only descend by driving it out through *o* (the only opening) into C. Also, C being full, water can only enter by driving out P, *i. e.*, by pushing it up, and

with it T. So that it becomes a question whether P and T shall rise, whatever may be the weight on them, or whether the machinery shall give way at some point or other. The water being all but incompressible, being

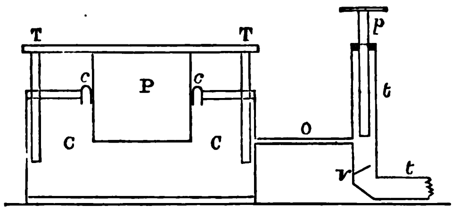


Fig. 82.

practically quite incompressible, cannot give way. So that if water can be forced in—*i.e.*, if the machinery be strong enough—P and T must rise, even if tens or hundreds of tons weight be resting upon them.

It will be seen that *p* being so small in proportion to C, but little water can be forced in at a time, so that P will rise but slowly; still, it does rise, and can be made to rise any height, within the length of P. The slow but steady rise of T renders the machinery specially suited to the moving of very large bodies, such as portions of bridges, hulls of ships, &c., no weight being too great for it, provided the machinery be strong enough to bear the pressure.

But water, besides being incompressible, is practically infinitely divisible—that is, it readily divides into excessively small pieces resembling dew. To prevent the water from oozing out round P, a leathern collar was invented by Mr. Bramah (whence the hydraulic press is often called the Bramah press). This is shown in fig. 82 by *c c*, as being curved, with the edges bending down; and the result of this is that the greater the pressure of the water the more tightly does the collar fit, being pressed against the edge of the opening and against P more and more tightly as the pressure of water increases.

This invention was the commencement of the use of the hydraulic press for very great weights.

The extent to which P rises depends upon its size. The larger it is the more room does it give in moving, and therefore the less distance will it move as the result of the application of any given force at p. If P be a hundred times as large as p, it will move $\frac{1}{100}$ of the distance through which p moves—being one more example of the truth of the theorem that machinery cannot create power, but only apply it advantageously.

65. The Inclined Plane.—Two unequal weights may be made to balance each other by being made to act at different angles to the beam of a balance, as in fig. 83. This is quite independent of the length of A B. So that it may be shortened to any extent without affecting the truth of the theorem. In fact, A and B might coincide, and then we should have two inclined planes in contact.

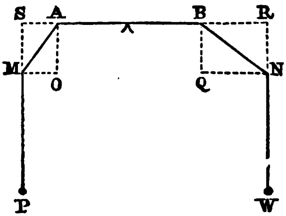


Fig. 83.

Again, the inclination of these two to each other might be anything between 0° and 180°; and the two forces, P and W, might act upon each other at any angle between hanging parallel and being in the same straight line.

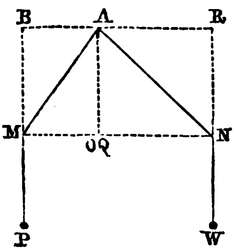


Fig. 84.

It is quite clear from this that a weight on an inclined plane will be balanced by a weight less than itself that acts vertically. The lever, A B (fig. 83), may be shortened, until A and B coincide, and then we have a weight, W, supported on an inclined plane

by another, P, acting vertically. But P need not be a weight hanging by a cord.

A body on an inclined plane may be supported either from above or from below. In fact, by shortening A B, and then shortening the string, N W, we come at last to the simple case of a body on an inclined plane, kept in its place either by pulling from above or pushing from below.

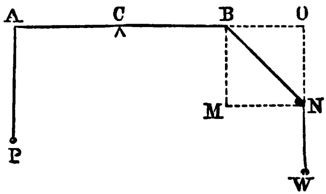


Fig 85.

We have now to ask what are the conditions under which the body may be kept on the inclined plane? What ratio must the power have to the weight before it can move it? What influence will the degree of inclination have upon this ratio?

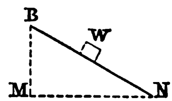


Fig. 86.

The line B N being taken to represent the actual weight of W, then the line B M will represent its vertical force, and therefore also the amount of vertical force that will balance it. Thus, if the angle B N M be 30°, then B N = 2 B M, and 1 ℔ hanging vertically at B, will balance 2 ℔s resting on B N.

That is, the conditions of equilibrium upon an inclined plane are, that the power and weight shall be in the ratio of the height of the plane to its length. To move W, resting on the slope, P must have a greater ratio than this: thus if, as before, B N M = 30, then if $P = \frac{1}{2}$ W, the latter will remain at rest; but if P be greater than $\frac{1}{2}$ W, the latter will be pulled up the plane by the former.

Finally, to answer the question, What influence will the degree of inclination have upon the ratio of P to W? it is evident that the greater the inclination of the plane, the more readily will W slide down it, and the greater will be the force required to keep it at rest.

Thus, if 1 ℔ hanging from the top of the plane just keeps at rest 3 ℔s resting on it, then, if the plane be raised at B so as to have a greater slope, the 1 ℔ will cease to be sufficient, and the 3 ℔s weight will descend, raising the 1 ℔ by its fall.

66. The Screw.—An inclined plane may be wound round an axis as a thread is wound round a reel. It ceases to be a plane, and becomes a screw; but the conditions of equilibrium, the ratio of power to weight, and the influence of increase or decrease of inclination, all remain exactly as before. The inclination of the plane now becomes the distance of the threads of the screw from each other. P becomes the force required to turn the screw round, and W the weight which has to be moved before the screw can be turned.

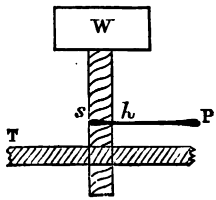


Fig. 87.

If a weight, W, be raised by means of the screw, S, moved round by the handle or lever, P; when the screw is turned right round once, W is raised through the distance of one thread of the screw from another.

This one thread of the screw might be unwound from its axis, and would then be an inclined plane, A B C: the distance, B C, would represent the distance through which W had been raised by one turn of the screw, and A B would represent the inclined plane up which it had travelled.

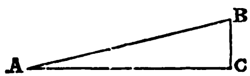


Fig. 88.

Now, how has P been applied in order to raise W? By means of a handle, *h*, and the force travels in a circle, of which the length of the handle is the radius. We come here again to our old theorem: a small force

moving through a greater distance will move a larger weight through a less distance. The longer the handle, *h*, the less force will be necessary; but it will move round a larger circle. Let $h = 2$ feet; then the circumference of the circle of which it is the radius is about 12 feet 6 inches.

Then W : P : : $12\frac{1}{2}$ feet : B C = distance between two threads of the screw.

Let B C be one inch, and W = 100 lbs.

Then W : P : : 150 : 1.

or 100 : P : : 150 : 1.

$$\therefore \; P = \frac{100 \times 1}{150} = \tfrac{2}{3} \text{ lb.} = 11 \text{ ounces nearly.}$$

That is, a force of 11 ounces acting at the end of a lever 2 feet long will suffice to counterbalance a weight of 100 lbs. The 100 lbs would not be able to push the screw down, because of the resistance of the 11 ounces: the 11 ounces would not suffice to raise the screw, because of the resistance of the 100 lbs. Increase either of these, and motion would follow. But for every turn of the screw—that is, for every inch that W is raised—P moves through 150 inches; and therefore, for equilibrium to exist, $P = \frac{W}{150}$.

67. The Wedge.—If I push an inclined plane under any weight, it is raised gradually by the pressure of the plane upon its under surface. If I put two inclined planes together, base to base, I have a wedge, A B C. This driven between any two bodies would separate them in the same way. If I chop wood, the axe acts as a wedge; so does a knife in any ordinary cutting operation; so does a nail which is driven into wood. But the knife or axe is often wedged in by the cohesive force of the substance being divided; the nail is usually fixed where it is driven; and there would seem to be here cases of equilibrium after the applied power had ceased to act. A

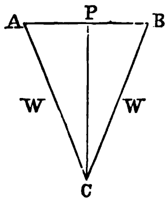


Fig. 89.

nail is kept in its place by friction, not by a weight resting on its head. So a wedge driven into a log of wood is held fast by the cohesive force of the timber, unless the force used in driving it has been sufficient to overcome this, and to divide the wood completely.

It is, however, necessary that the force used in driving the wedge should be more than sufficient to overcome the cohesiveness of the wood to a certain extent, otherwise the wedge would not be driven in at all. So, though we cannot form a simple expression of equilibrium between P and W in the case of the wedge, we may regard a wedge as a double inclined plane; the pressure of the wood upon each face of it may be considered as a kind of weight to be raised by it, and the force applied to drive the wedge in may be considered as a power. Unless this power be greater than the two weights (*i. e.*, the cohesive force of the two parts of the wood to be divided), the wedge cannot be driven in.

Evidently the thicker the wedge, the more force will be required to drive it; so that there is some relation between A B and P C on one side, and between P and W.

$$\text{As } AB : PC :: P : W$$

might seem at first to be the formula to express this; but the relation between P and W differs, in the case of the wedge, from others, such as the lever or the inclined plane, inasmuch as force applied to the wedge usually produces a permanent result (the wedge remaining where it is driven); while in other cases the force expressed by P is only effective while it is actually being applied.

68. **The Pulley.**—If I suspend a stone weighing 100 ℔s by four cords, one at each corner, it is evident that the whole weight will be supported by the four cords, and that the strain on each of the four will be the same;

therefore each cord will support a weight of 25 lbs. This will be the same whether all the four cords are fastened to the same point of suspension, H, or to four different points, A. It will also be the same whether the cords terminate at the point or points of suspension, H or A, or whether they continue beyond. It is also true, whatever the number of cords to which a weight is suspended, the total weight will be divided equally between all the cords, whatever their number and whatever their respective degrees of strength. Provided the weakest were strong enough to bear its quotient of weight, it would have as great a strain on it as the strongest.

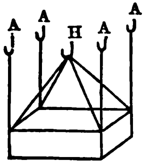


Fig. 90.

One cord alone might be used to sustain a weight which, suspended in the usual way, would suffice to break it. Thus, I take a cord that will sustain only a weight of 100 lbs, and, fastening it to a hook, pass it through the ring of a weight, W, of 400 lbs, again over the hook, and again through the ring. There are now three folds of string, and by passing it again over the hook I have four, each one capable of bearing a strain of 100 lbs, and the whole four able to bear W, which is 400 lbs. The end of the rope, P, may be held by the hand, or a weight of 100 lbs may be fastened to it. In either case the weight of 400 lbs will be balanced by one of 100 lbs.

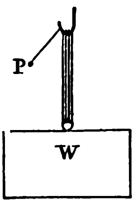


Fig. 91.

Not considering the effect of friction, a weight of 100 lbs would cause the weight of 400 lbs to rise; but (as in all other cases) the small weight will descend through a greater distance than that through which the large weight will rise. If I draw P down 4 feet, each fold of the string between the hook and W will be shortened but 1 foot, for there is but one cord throughout. Conversely, if I were to raise the weight, W, through 4 feet,

each of the four cords would be shortened 4 feet, and the end, P, would descend through 16 feet.

I may, therefore, in this way raise large weights by means of a small power moving through greater distances; but if I simply pass a rope several times over a hook, the friction will be so great that probably no motion will result, as the greater the strain upon any fold of the rope, the greater will be the force with which it will press upon any of the folds that may be beneath it. To avoid this, I may use small rollers to diminish the friction of the rope when passing, and also to keep the cords distinct, using one roller to each cord.

A roller of this kind is called a "pulley," and a number of them arranged in a frame is called a "system of pullies."

A pulley being a roller of wood or metal: I fasten one of these, *a*, to the ceiling, and pass the rope fastened to W over it. What weight at P will balance 100 ℔s at W? Evidently it will require 100 ℔s: and the use of the pulley *a* will give no advantage, except that of being able to raise W by pulling down P, instead of having to raise it directly from the ground.

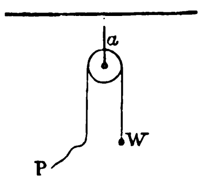


Fig. 92.

But if, instead of fastening the pulley to the ceiling, I fasten it to W, and the rope to the ceiling, I shall be able to raise 100 ℔s at W, by means of a little more than 50 ℔s at P (fig. 93). For if I raise W from the ground by means of force applied at P, it will be supported by the two parts, *r*, *s*, of the rope, and each of these will support 50 ℔s. Con-

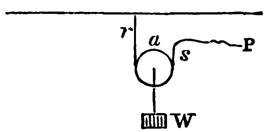


Fig. 93.

sequently 50 lbs at P will suffice to balance 100 at W, and any force above that will raise it. Notice that, in the arrangement shown in fig. 92, P and W are equal, and move equal distances; while in that of fig. 93 P moves 2 feet for every 1 foot that W moves, but is only one-half of W.

Any weight, W, can be supported by a weight, P, one-half of W, by passing a rope, *r*, round a pulley, *a*, fastened to W. But instead of applying P directly to the rope *r*, we may use a second pulley, *b*, round which a second rope, *s*, passes, and even a third pulley, *c*, and a third rope, *t*.

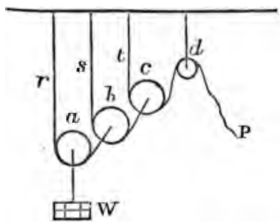


Fig. 94.

Let W be 100 lbs, then the rope *r* has a strain of 50 lb on it, and this strain is supported by the rope *s*. But this rope supports the pulley, *b*, by passing round it, so that the strain upon each part of this rope is 25 lbs, which is supported by the rope *t*, passing round the pulley, *c*, so that the force required at P is only 12½ lbs. For convenience, the final rope may pass over the small pulley, *d*, which will not affect it otherwise than by changing its direction.

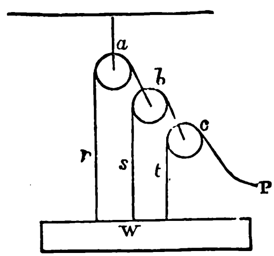


Fig. 95.

The same arrangement may be reversed, as in fig. 95, by fastening all the ropes to W, and one pulley only to the ceiling. Here we shall not need the little pulley, *d*, to

enable us to apply the power P, by pulling, as the rope already comes downwards from the pulley *c*.

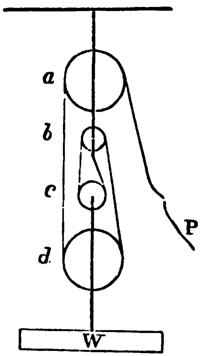


Fig. 96.

Finally, the pulleys may be grouped more compactly, as in fig. 96, in which one rope suffices for the whole. Here the upper pulleys, *a b*, serve only to enable the cord to come down again, while the lower ones, *c, d,* support the weight W, and enable a small force, P, to balance it. Thus the rope passes round two pulleys, *c d,* and W is supported by four ropes, so that the strain at $P = \frac{W}{4} = 25$ lbs (if W = 100 lbs).

69. Resultant of Parallel Forces.—At page 27 I have shown that the resultant of two parallel forces is parallel to them and equal to their sum; and I said that the direction and amount of this resultant could also be found, geometrically, by the use of an artifice which I will now explain.

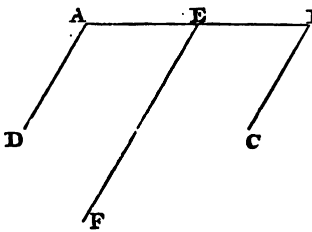


Fig. 97.

Let there be two forces, A and B, acting parallel to each other along A D and B C: then we cannot make them two adjacent sides of a parallelogram. But we may add two equal forces, one to A and one to B, without affecting the result.

Thus to A D add A M, and to B C add B N, acting at A and B along the line A B, and in opposite directions; they will counteract each other, and so not affect

the direction and magnitude of the resultant of A D and B C; but they will also enable us to get two lines, S A and T B, which will converge and meet at O. These two lines represent the resultants of A D and A M, and of B N and B C: these, not being parallel, will meet. From O, the point of meeting, draw O F, parallel to A D and B C, cutting A B at E. This gives the direction of the resultant of A D and B C: its magnitude is (as before) equal to A D + B C, since no part of either is lost.

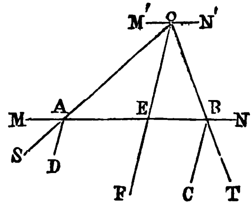


Fig. 98.

We can now determine, geometrically, the point E, through which O F passes. As we have seen (page 27), this is the middle point of A B when A D and B C are equal.

Generally, A E : B E :: B C : A D,
which is the same as before, and corresponds with the lever. To prove this, we have the triangle, A E O, formed of three sides, representing in magnitude, and parallel in direction to, the two forces A D and A M, being represented by O E and A E, and the resultant A S by A O. Therefore, the three sides of the triangle are in the same ratio to each other as the three forces: and therefore,

A E : E O :: A M : A D.

By the same reasoning, we have, in the triangle, B E O,

B E : E O :: B N : B C.

Compounding these two, we have,

A E : B E :: B C : A D.

That is, the point E divides A B into two parts, A E and B E; so that A E is to B E as the force B C is to the force A D.

But the parallel forces may act in different directions: thus, A D from A to D, and B C from B to C. Will our geometrical construction and proof help us in this case also? Let us follow it out rigorously:—Let B C be the greater force: add A M and B N, two equal lines, to A and B

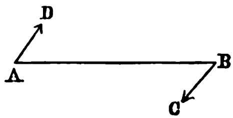


Fig. 99.

respectively, draw A S and B T (the resultants of A M, A D, and of B C, B N), produce those lines to meet at O,

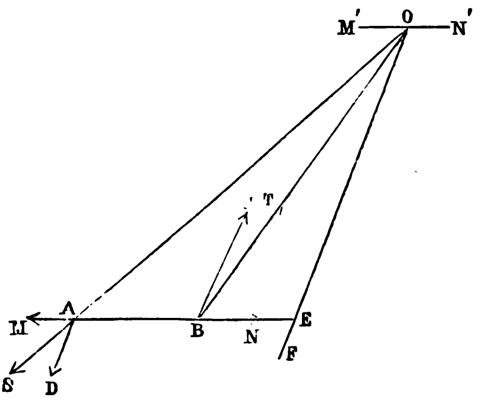


Fig. 100.

and draw O F parallel to A D and B C. This line will not cut A B at all, as it will be beyond B; but if A B be produced indefinitely beyond B, the line O F will cut it at E, and E F will be the resultant of A D and B C, and E will be the point through which it will act. What will be its magnitude? When the forces were parallel and acting on the same side of A B, the resultant was equal to their sum—*i.e.*, E F = A D + B C. Now that they are parallel, and act on opposite sides, they

counteract each other so far as they exist together, and if they be equal, the resultant is nothing; but if one be greater than the other, then it is equal to their difference—*i.e.*, E F = A D − B C, if A D be the greater; or E F = B C − A D, if B be the greater.

But it may be asked, How do we know that O F, drawn from O, is the right line for the resultant?

To explain this we must recal what has been said about the resolution of forces (page 25), where it has been explained that if the force A D be known, and its components be required, they are found to be any two that will form a parallelogram, of which A D is the diagonal. From this we saw also, that if A D and A B were given, A C could be found; or if A D and A C were given, A B could be found.

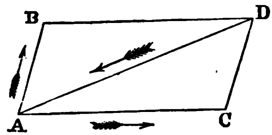


Fig. 101.

Now, in fig. 102, we have an example of this resolution of one force into two: the force A S is compounded of the two forces, A D and A M, A being the point of its application. But it may be considered to act at O (A O being rigidly connected with A S). At O, then, we have a force acting along O A equal to A S. Let us resolve this into two forces: we can do this without limitation, other than that the two lines must make a parallelogram, of which as much of O A as will = A S shall be the diagonal. Let one of these two forces be O M′, equal to A M, and parallel to it. Then the other must be equal and parallel to A D, acting at O—*i.e.*, it will be along O F.

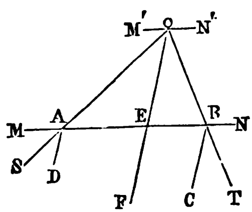


Fig. 102.

In the same way, B T (which is compounded of B C and B N) may be transferred to O, and these resolved

into O N' (which is equal and parallel to B N), and along O F, equal to B C. So that both A D and B C are acting at O, along O F, and are therefore together equal to A D + B C.

In the second case, where the two parallel forces are

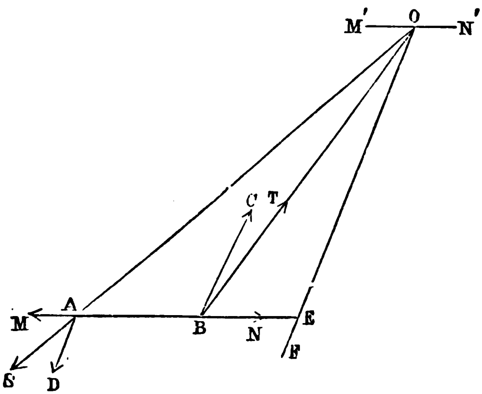


Fig. 103.

unlike, we have precisely the same method and the same result. The resultant A S (compounded of A M and A D) is considered to act at O (along O S), and to be there resolved into O M' (which is equal and parallel to A M), and a second force along O E, which is equal and parallel to A D. In this way A D is theoretically transferred to O, acting along O E. Likewise B T (compounded of B C and B N) is considered to act at O (along B O, not O B), and to be there resolved into O N' (which is equal and parallel to B N), and a second force along E O, which is equal and parallel to B C. Thus we have at O two forces, A D and B C, acting in different directions along the same straight line. If they be equal they counteract each other, and the result is O; if they be unequal, one partly counteracts the other, and

the result is A D − B C, or B C − A D, according to whether A D or B C be the larger. In any case, O E is the direction of the resultant, and its magnitude is the difference of the two unequal and parallel forces acting in different directions. Also, as before—

A E : B E : : B C : A D.
In fig. 102, let A D = 3 and B C = 2;
then B E will = $\frac{3}{5}$ of A B, and A E = $\frac{2}{5}$ of A B.
In fig. 103, let A D = 3 and B C = 2;
then B E will = $\frac{3}{5}$ of A B, and A E = $\frac{2}{5}$ of A B.

These seem alike in the two cases; but in the one case E is between A and B; in the other case it is not: in the one case A B is measured direct through E; in the other it is measured from A through B to E, and then back to B.

In each case E is the fulcrum round which the three forces balance. In one case (fig. 102) A D and B C, acting together, are counteracted by a single force equal to their sum acting at E, along F E; in the other case A D and B C (fig. 103), acting in opposition, are counteracted by a force, equal to their difference, acting at E, along E F, which is in the same direction as A D, the smaller of the two.

In the case of two equal and parallel forces acting together (fig. 98), the point E is midway between A and B; in the case of two equal and parallel forces acting against each other (fig. 100), it will be found that A O and B O are parallel, and will therefore never meet, so that the point E is infinitely distant. It would be necessary in this case that A E should equal B E, which is impossible. But the more distantly E is removed from B the nearer does B C approach to an equality with A B, and when it is infinitely removed they may be considered to be equal, but then the point E cannot be found.

70. The Differential Axle.—In the ordinary use of a pulley, or system of pulleys, the end of the rope is drawn down in order to raise the weight; that is, for every foot through which W is raised, P is lowered 2, 4, 6, 8, or 10 feet, according as W is supported by 2, 4, 6, 8, or 10 lengths of rope.

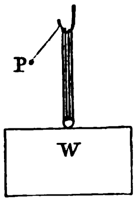


Fig. 104.

The same result may, however, be attained by a rotary motion of P; that is, the weight W may be raised by turning a handle. In fig. 105 is shown an arrangement for this purpose, which is much more compact and neat than a group of pulleys with a long rope to be drawn down through many feet. Two axles, one, A, larger than the other, B, are turned together by one handle, H. Now, if a rope be wound round the large axle, A, and the end be fastened to B, it is evident that as it is unwound off the one axle, A, it will be wound on B in the contrary way. But B, being smaller than A, will not take up all the rope unwound off A, so that some of it will be left loose. Thus supposing the circumference of A to be 15 inches, and that of B only 8 inches; then, for every 15 inches of rope unwound from A, 8 inches will be wound upon B, and 7 inches will remain loose: for 20 revolutions 300 inches will be unwound from A, and 160 inches wound upon B, leaving 140 inches, or nearly 12 feet, hanging from the axles. Now, if a weight, W, be suspended from this rope by a pulley, it will descend as the rope is wound off A, and rise as it is rewound on it.

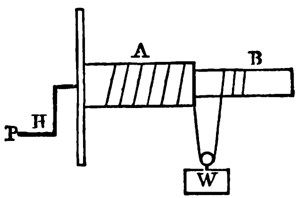


Fig. 105.

The weight, W, tends to unwind the rope from the two axles; but to unwind it off one is to wind it on the other, so that the weight of W pulls the rope two different

ways. But the effect upon A is greater than the effect upon B, because the radius of A is greater than the radius of B, and the radius is the length of the lever at the end of which W acts upon the axis (or centre) of the axle. The weight will therefore descend of itself, unless the friction be sufficient to counteract the tendency of W to unwind the rope from A.

This tendency to descend on the part of W has to be counterbalanced by a force, P, applied at the handle, H; and by an increase of this force above what is required to balance W, the latter can be made to ascend. We have three forces at work, two acting together, and the third acting against these two.

Thus, if W A express the moment of W about the axis of the greater axis (*i.e.*, its tendency to unwind the rope from it), and if W B express the opposite tendency to wind the rope off the axle B, then, if the handle be turned so as to raise the weight W, we have

$$P\,H \text{ is greater than } W\,A - W\,B:$$

and for equilibrium we have

$$P\,H = W\,A - W\,B$$
$$\text{and } P\,H + W\,B = W\,A.$$

H is the length of the handle, at the end of which P acts; A and B are the radii of the two axles. But the whole weight cannot act both on A and on B. Since W is supported by a double rope, the weight (or tension) on each half of the rope is half of W, *i.e.*, $\frac{W}{2}$, and then we get the above equation changed to

$$P\,H + \frac{W}{2}\,B = \frac{W}{2}\,A$$
$$\therefore 2\,P\,H + W\,B = W\,A$$
$$\therefore 2\,P\,H = W\,A - W\,B$$
$$\therefore 2\,P\,H = W\,(A - B)$$
$$\therefore P = \frac{W\,(A - B)}{2\,H}$$
$$\therefore \frac{P}{W} = \frac{A - B}{2\,H}:$$

and this may be changed to

$$\frac{W}{P}=\frac{2\,H}{A-B};$$

that is, $W : P :: 2\,H : A-B$;
or W is to P as twice the length of the handle is to the difference between A and B—that is, between the radius of A and the radius of B.

This will show us how to adapt the machine either to raise a small weight or a large one. The greater the difference between the two axles (or rather the two parts of the same axle) the more rapidly will W rise or descend; and, therefore, to raise a small weight quickly we should use an arrangement in which the difference between the two diameters should be as great as consistent with strength and convenience.

To raise a large weight the two diameters are made more nearly the same, so that the weight rises slowly. There is no limit to the weight that could be raised by a machine of this kind, except the limitation of strength in the apparatus.

But it is true with this, as with all other machinery, that what is gained in power is lost in distance or in time—that is, P has to move through a greater distance to move W through a less.

The equation $\frac{W}{P}=\frac{2\,H}{A-B}$ will give us the relation between the weight and the power necessary to move it. If the length of the lever, H, at the end of which is the handle, be 12 inches, and the difference of the radii of A and B be 1 inch, then 1 ℔ at H will move 24 ℔s at B.

In one revolution of the handle, P will move through $24 \times 3{\cdot}1416 = 75{\cdot}3984$, and W will move through 3·1416 inches—*i.e.*, through $\frac{1}{24}$ of the distance, for 2 inches by $3{\cdot}1416 = 72832$, which is the length of rope left loose at every revolution. But the weight is supported by the doubled rope, and therefore rises or falls through only half the length of the loosened rope.

The equation $\frac{W}{P} = \frac{2\,H}{A - B}$ becomes in this case $\frac{24}{1} = \frac{2 \times 12}{1}$, for the difference of the two diameters being 2, the difference of the radii will be 1.

71. Compound Levers.—In describing the aneroid barometer (page 81), I said that the effect of the pressure of the air on the metal box was multiplied by the action of compound levers. A simple example of these is shown in fig. 106, where A B, B C and C D are three levers, with fulcrums, *a*, *b*, and *c*.

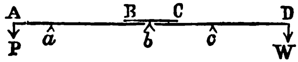


Fig. 106.

Let B *a* be twice *a* A, then B is moved twice as far as A; if B *b* and *b* C be equal, this motion is transferred unchanged in amount to C. Finally, if D *c* be twice *c* C, the effect is again doubled at D. So that for every inch that A moves, D moves four inches.

The advantages of this system are—1st, that motion can be transferred in a curved line as easily as in a straight line, for there is no reason why the levers need be in the same straight line; 2nd, the levers being shorter, are not liable to bend so much with any given force.

The levers may be arranged so as to form curved lines, either because a straight line arrangement is not convenient, or because it is required to bring P and W close together, or because it is required to transfer force through a series of points that are not in a straight line, as in fig. 107, in which we are supposed to look down upon the lever, where force may be transferred from A to D, along the lines A B, B C, and C D, passing through the points B and C. If all the fulcra, *a*, *b*, and *c*, be at the middle points of their respective levers, there will be the same amount

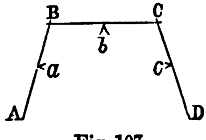


Fig. 107.

of motion at D as at A, but in a different direction, one being up and the other down. But, by varying the position of the fulcrum in each lever, we may have a change of amount as well as of direction of motion.

Thus, if *a*, *b*, and *c* be so placed that *a* B be 10 times *a* A, also *b* C be 10 times *b* B, and *c* D be 10 times *c* C, then we shall have the amount of motion at A increased tenfold at B, a hundredfold at C, and a thousandfold at D. So that 1 ℔ at D will balance 1,000 ℔s at A; or, *vice versa*, it will require 1,000 ℔s at A to balance 1 ℔ at D.

In this way, by the use of a series of compound levers, heavy weights, such as loaded waggons, and railway trucks, are weighed.

72. **Friction.**—In all that I have said about the effect of a force in producing a preventing motion, I have always assumed that no part of it was lost. But, practically, this is impossible; no two surfaces are so smooth that no loss of power occurs when they pass over each other. In theory it is necessary to leave this fact entirely out of consideration, and to assume that all the machinery of lever, pulleys, wheels, &c., work without friction in the same way that we assume them to be without weight. Otherwise we could not obtain any general formulæ or laws.

But friction is a very important matter in actual working of machinery, and must be taken into consideration, as also must the various means of diminishing its effects. As an example of how much friction diminishes the practical effect of any force, and also to show how, by various artifices, it may be counteracted, the following set of experiments will be interesting:—

A rough stone, weighing 1,080 ℔s, required a force of 750 ℔s to drag it along the rough floor of the quarry, but only a force of 650 ℔s to draw it along a wooden floor, made by laying down planks on the quarry floor. The stone itself, being placed on a wooden platform, was drawn over the wooden flooring by a force of 600 ℔s, and the greatly decreased force of 182 ℔s sufficed when

the two wooden surfaces in contact were smoothened by being well smeared with soap.

So far the experiments were by simply drawing one surface over another. A second set was tried with rollers. Rollers of 3 inches diameter were placed under the rough block of stone, and it was drawn over the rough quarry floor by a force of 34 lbs, while 28 lbs sufficed when the wooden planks were laid down under the rollers; and, finally, 22 lbs was enough to draw the same stone, weighing 1,080 lbs, when mounted on the wooden platform and moved upon the 3-inch rollers over the floor of wooden planks.

When we say that a road is rough, we mean that it has numerous projections: to draw any body along such a road, it has to be lifted over these projections, or else drawn with such force as to remove them out of the way. By using wheels or rollers, this difficulty is greatly lessened; for the wheels (which themselves become the moving body) are not drawn across the surface, but roll over it—do not have to be lifted up over each projection, but rather travel over their little summits. In fact, a roller passing over a rough road may be considered as being a cog-wheel travelling along a rack, the teeth of the wheel and the rack being very small, and not arranged regularly.

73. Motion.—By motion is meant that a body changes its position, gradually, with relation to other bodies. Walking, rolling, dragging, pushing, are varieties of the manner in which motion can take place; but depend rather upon the aggregation of a number of particles, since a single particle cannot well be either rolled, dragged, or pushed, still less can it walk.

It is necessary to consider motion as of a single particle in order to clear it of all its accessories, and then we may be able to realize that motion is simply a continuous change of position, with relation to other bodies, that are assumed to be stationary. This last is very important; for, unless we have some point which we take as fixed, it is impossible to say whether or no any given

body be really moving. For example, any one sitting in a railway carriage, with a train in motion on each side of him, will be quite unable to tell whether his own carriage be moving or no, so long as he looks only at the moving trains beside him. Even when we compare our motion with a fixed point, it is not always easy to tell even in which direction we are moving. Most people are familiar with the frequent delusion as to the direction in which a train is moving when we are going through a tunnel, or when the ground in view is alike for some distance. If there be posts, gates, or any definite objects visible as we pass, then we are in no doubt as to our direction; but if there be an embankment, of which every succeeding yard is like the preceding, it is very common to be mistaken as to the direction in which the train is moving, so far as our external impressions are concerned.

This is probably to be explained by the variations in the speed of the train: if this be decreased gradually, we are literally going in a different direction to that of the train. It is a common occurrence for the driver of a cart to be thrown to the ground if the horse fall, and he usually passes over the horse and falls in front of him. He is travelling at a certain rate, his weight being supported by the seat of the cart. This suddenly comes to rest, while he passes on, and his weight being unsupported, he falls to the ground. Had the cart come to a stand gradually, the friction between the man and the seat would have brought him to rest with it. In the same way, if the speed of a train be reduced, we travel more slowly with it; but we have the sensation of being carried back, and in fact are being carried back. If we were suddenly set quite free from the train, and gravitation did not act to pull us down, we should go on in advance of the train at the rate at which we had been travelling. The friction within the seat of the carriage compels us to slacken our pace with the train, and the practical result is that we are really being held back. We feel this; and if there be no distinct objects visible

as we pass them, we have the sensation of travelling the reverse way to that in which we know we are moving. If there be fixed objects of definite shape within view, their apparent motion gives us an impression of motion in the way in which we are really moving.

The importance of fixed objects that are near, as a means of judging of the rate of motion, may be realized by looking at a cab, a man, or a railway train, that is advancing directly towards us, when it will appear to be perfectly still. A train advancing to a platform in a straight line will appear to be stationary until it comes sufficiently near to pass some object quite close.

It is quite possible for a body, apparently in motion, to be really at rest, and for the objects which appear to be at rest with regard to it to be moving. Thus the earth travels round the sun, rolling eastward, at about one thousand miles an hour. If a railway train were travelling the reverse way, east to west, at precisely this rate, it would really be at rest, relatively to the sun, though moving relatively to the earth. But this motion with respect to the earth would be the result of the force moving the train, while its rest, as compared with the sun, is the result of the motion of the earth; still, if the two velocities were exactly equal, and the two directions exactly opposite, the train would really remain at rest.

So that, when speaking of motion, it must be clearly borne in mind that it is necessary to have some fixed point which is considered to be at rest. Motion by itself, independent of all relation to any other than the moving body, is impossible of conception. For its comprehension, its measurement, if not for the very notion of its existence, it is essential that a point independent of the moving body, and assumed to be at rest, shall be taken into account. It is indispensable, for the very idea of motion, that the point from which the moving body starts, and the point at which it arrives, should be estimated.

It is impossible to speak of motion without saying that

it passes from one place to another; it is impossible to describe or estimate it without saying it is at the rate of so many feet, or yards, or miles, in some unit of time. For the conception of motion we must think of some fixed point—*i.e.*, fixed as compared with the moving body. For the measurement of motion we must think of both space and time, and of a unit of each.

74. First Law of Motion.—*A body at rest remains at rest until some force acts upon it to set it in motion.*

A body in motion continues with its motion unchanged, either in direction or velocity, until acted upon by some external force.

The second proposition includes the first, for it is one limit of it, since motion may decrease in velocity until it becomes actual rest.

A circular plate of polished steel, fixed on an axis, and enclosed in a glass case, from which the air has been pumped out or otherwise exhausted, will, when once set revolving, continue in motion for a long time without even any apparent diminution of velocity. In this case the circumstances are favourable to continued motion, there being but little to retard it. The friction upon the axle is reduced to the smallest possible amount; the resistance of what little air there is in the case is confined to that resulting from friction; for the plate of metal being a circle, continues to occupy precisely the same space, so that no air has to be removed. Yet these two resistances—that of the axle and that of the air—both acting as friction, do eventually bring the plate to rest. So that we have two points to consider:—*1st.* That a body set moving will apparently continue moving for an indefinite period, unless brought to rest by some external force. *2nd.* That any resistance, however small in amount, will eventually bring a moving body to rest.

It is difficult to believe that a body when once set in motion would continue to move for ever; but it is equally difficult to believe the contrary, when we consider that our very existence, whether as individuals or as parts of the universe, depends from moment to moment

upon its being true. For unknown ages our globe has been circling round the sun, kept in its place by two forces—one the original force that set it moving, the other the attraction of the sun. Suppose either of these to cease, an immediate change would take place in our position. If gravitation were to cease acting between the sun and the earth, the latter would fly off at a tangent into space. The force that first set it moving would act upon it to propel it in a straight line; and as no other force would exist capable of altering this, the earth would continue to travel away from the orbit of the sun in a straight line. Suppose this force to cease to act, the other continuing, the attraction of the sun would draw the earth in a straight line to itself. So that in one case we should travel eternally away from the sun; in the other we should be drawn at once into contact with it. In either case our existence in our present condition would soon come to an end.

Since, then, this law of continued motion has kept the globe circling round the sun with undiminished and even unvarying speed through untold millions of years, it is even more difficult to disbelieve in its truth than it is to understand it. We have innumerable examples of it. Two marbles set rolling by any given force, one on a smooth, the other on a rough surface, will travel through very different distances; and the smoother the surface the greater is the distance through which any given force will propel a body; so that we may fairly assume that if it could be made absolutely smooth, or if the moving body could be supported against gravitation without friction, it would continue to move for ever.

One way of explaining this fact is to say that the body, once set moving, must go on moving until some external force brings it to rest, because it has no power of its own, no volition even, and can do nothing whatever towards arresting its career. If we remove all sources of obstruction, take away all obstacles, then either it must continue for ever in motion, or it must come to rest through the exertion of some power of its own. But it has no power

to exert: it is an inert mass, and therefore it moves until brought to rest by the exertion of some external force, equal and opposite to that which set it in motion.

There seems to be no reply to this; but it does not satisfy some minds. It has been said that we measure forces by their effects, and their effects by the distances through which they move bodies capable of being moved; that to suppose that every force confers perpetual motion is to suppose that all forces are equal; that we say, in describing a force, it is capable of producing motion in such a direction for such a distance; that we do not assume there are to be any opposing forces to bring the body to rest; that in our theorems of the triangle, parallelogram, and polygon of forces we assume that any given force has the power of producing motion to a certain distance (neither greater nor less), and that we can so implicitly depend upon the truth of this assumption, that we can build mathematically upon it—can base a vast edifice of mathematical reasoning upon it—as one of a number of never-failing supports; that in the same book we are told, 1*st*, that any given force will produce motion to a certain distance, all obstacles being removed; and, 2*nd*, that any body once set moving by any force will continue to move for ever, unless brought to rest by some other force.

To this it may be replied that, in the polygon of forces, time is an element of the proposition; that forces are measured, not by the absolute distances through which they can move bodies, but by the distances through which they can move them in given times. The greater of two forces will move a given weight through a greater distance in a given time than the lesser force, or it will move a greater weight through an equal distance. Two unequal forces may be assumed to be capable of moving any two bodies of equal weight, and of setting them in motion for ever; but at the end of any given time (however great or however small) the distances through which they will have moved will be proportional to the forces.

Also, it has been said that we speak of forces as being almost entities—almost things that we can add, and divide, and arrange, and re-arrange, but cannot destroy, as being definite in amount and effect. We measure them into "horse-powers" and "foot-pounds," speak of their convertibility into each other, and yet while speaking of them as being so definite, so capable of measurement, we say they produce effects which are altogether so indefinite, so illimitable. On the one hand, we say that a given amount of steam power will produce a certain definite amount of motion, and no more; on the other, that a body once set in motion, even by this very steam power, will continue for ever to move.

But in the one case we are speaking of machinery, in which the force applied and body to be moved are connected—one rising as the other falls, one going to the right as the other goes to the left. In the other, we speak of force and body to be moved as being independent of each other, after the body is once set in motion. In the one case we are speaking of levers, pulleys, inclined planes, on one side of which is the force, on the other the weight to be moved; of machinery, by means of which force is converted into motion, which is force under another name; of arrangements for utilizing, transferring, subdividing any given force. In the other, we are speaking of the amount of motion given to a mere particle moving in space, and perfectly free from any influence of any kind.

Thus a force of 100 ℔s applied to one end of a lever will move a force of 99 ℔s fastened to the other end, if the arms be equal; a force of 199 ℔s at the end of an arm twice as long as that at which the 100 ℔s act. If we move the 100 ℔s through one foot, the 99 ℔s will move through the same distance, or the 199 ℔s through twice the distance. The coming to rest of one arm necessitates the coming to rest of the other. But suppose, instead of a weight fastened to the other end, we have a ball resting on it, as in the case of a trap and ball, then a force applied to the other arm would set the ball in

motion; and if there were no friction with the air, no gravitation to draw it down, it would continue to move for ever.

75. Second Law of Motion.—*Every force acting on a body, whether in motion or at rest, produces its full effect of motion.*

In the parallelogram of forces (page 19), we assume each of two forces to act as if by itself, and find the resultant by taking the diagonal of the parallelogram, of which their directions form the sides. In the polygon of forces (page 23) we assume each force to act by itself, but also that the forces act in succession, each commencing where and when the effect of its predecessor came to an end. This last would seem to be most in accordance with the law here expressed, that every force produces its full effect of motion; but the essence of the theory of the composition of forces requires that the forces shall act at the same time. The former, in which the true line of action is found by forming a parallelogram and taking its diagonal, is the one in accordance with fact; but does not seem to be in accordance with the law, since the two forces, instead of each producing its full effect, produce a joint result, which is less than the sum of their separate effects. Just as it requires less labour to walk along one side of a triangular space than along the other two sides, so less force would move any given weight along the diagonal of a parallelogram than along two of its adjacent sides. Therefore it would seem that two forces acting together do not produce their full effects, though they produce a result that is practically the same.

But it must be remembered that the forces acting upon any given body may themselves be the resultants of other forces, and that even if not so, they may be so considered. Thus, in the case of two forces, A B and A C acting at A, the line of motion would be A D, the resultant of the parallelogram, A B D C. But A B may be the resultant of two forces, A E, A G; and A C the resultant of two forces, A F, A H. Supposing this to

be so, then the two forces, A E and A F, being equal and opposite, would counterbalance each other, while the forces, A G and A H, would act together with their full force.

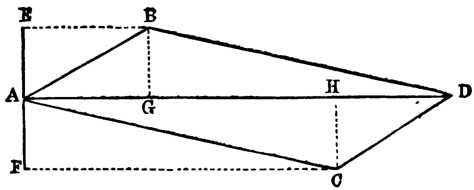


Fig. 108.

For this to be accepted, we want three things to be proved:—*1st.* That any two forces acting together may be resolved into two forces each, one of each pair acting along the diagonal of the parallelogram, of which the lines representing the forces are two adjacent sides, the other acting at right angles to this diagonal.

That is, the forces A B and A C may be resolved into two forces each, A B into A E and A G; and A C into A F and A H—A G and A H being along the diagonal, A D, and A E and A F at right angles to it.

2nd. The two forces acting along the diagonal shall be together equal to it—that is, $A\,G + A\,H = A\,D$.

3rd. The two forces, acting at right angles to the diagonal, shall be equal, and shall therefore counterbalance each other.

The first is in accordance with the theorem of the "resolution of forces" (page 24), which is, that any force acting at a given point may be considered as the resultant of any number of forces which can be represented by lines making, with the line of the resultant, a polygon.

The second may easily be proved. In the triangles, A B G and C H D, we have the angles respectively equal, for $B\,A\,G = C\,D\,H$, $A\,B\,G = D\,C\,H$, and A G B

= D H C. Also, the side C D = the side A B. Therefore A G = H D, and B G = C H.

So that the two forces, A G and A H, are equal to a force, A D; for A H+A G = A H+H D, since H D = A G.

The third is also easily made clear. Since B G = C H, A E must also equal A F; for A E = B G, and A F = H C, each pair being the opposite sides of a parallelogram.

So that though the resultant of two forces is not equal to the sum of the forces, it is equal to their combined effective force, acting in the same line.

But it may still be said, "Is this true in the case of the two forces, A E and A F, where no motion whatever results—where, so far from each force producing its full effect of motion, each practically destroys the other?" It must be remembered that in theory we speak of particles, while in practice we deal with compound bodies. If a ball were struck by two equal and opposite forces, it would remain at rest, as a ball; but there would be in it a considerable amount of motion amongst the atoms of which it must be considered as a collection, it would be compressed, and there would be an amount of vibration amongst the particles composing it which would be the motion resulting from the application of the two forces. Just as a ball in a vice could not move as a whole, but would have its particles re-arranged by the pressure, so any body, struck by two equal and opposing forces, does not move as a whole—the force acting upon it is spent in the motion of its particles, which motion may be expansion, or it may be heat—this last being the name given to a motion of vibration only. And we may confidently say that where the application of a force does not result in motion of the body struck as a whole, it does result in motion amongst its particles, which motion is called heat. A body when struck must either move or become warm—most probably it will do both, part of the force resulting in motion, part resulting in heat.

SUMMARY.

Measurement of Time.—We measure time by years, days, hours, minutes, and seconds. The fixed unit of time is the time which the earth takes to go four times round the sun = 1461 days nearly. This time we divide into four years—one of 366 days (leap year), and three of 365 days. These days have an average length of 24 hours, or rather we divide the period of their average length into twenty-four equal parts, called hours: each hour is divided into 60 minutes, and each minute into 60 seconds. (Page 9.)

Measurement of Space.—The English unit of space is a yard, which is very nearly the length of a pendulum that vibrates once every second. Such a pendulum must be 39·14 inches; and, if the bob be 6·28 inches across, the rod of the pendulum would be exactly 36 inches. The French unit of space is a metre, which is $\frac{1}{40,000,000}$ of the earth's meridian measured through the poles. A metre is 39·384 inches. (Page 10.)

Velocity.—The rate at which a moving body passes from one point of space to another is called its velocity. The rate of one foot per second is the English unit of velocity; and in France the unit is one metre per second (page 11). Velocity is *uniform* when it is neither accelerated nor retarded throughout any given distance. If a moving body moves more rapidly than before, its velocity is said to be accelerated; if more slowly, to be retarded. (Page 12.)

Quantity of Matter.—A heavier body is said to contain a greater number of ultimate particles than a lighter. The more closely the particles of any given body are compressed, the greater is the quantity of matter in any given volume. (Page 13).

Momentum.—The force which a moving body can

impart to any other body is said to be its momentum. The greater its mass, the greater its momentum; also, the greater its velocity, the greater its momentum. (Page 14.)

Force.—The primary cause of motion we call force. What it is we cannot say; but motion cannot take place without the application of force. We cannot describe it otherwise than by its results. (Page 14.)

Unit of Force.—The power of moving one pound weight through one foot of space against gravitation (*i. e.*, to raise it vertically) is the unit of force. (Page 15).

Description of a Force.—A force is described by saying that it is capable of moving a given body a certain distance in a certain time. It is customary to describe this by drawing a straight line, whose direction indicates the direction in which the force acts, or tends to act, and its length the extent of the force, being drawn to some known scale. (Page 16.)

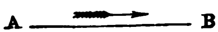


Fig. 109.

Composition of Forces.—If two forces act at the same time upon any given body, each produces its full effect in moving the body, and the line of motion is between the lines in which the forces act. Thus, if two forces, A B and A C, act simultaneously at A, the body on which they act will move in a line between A B and A C. (Page 17.)

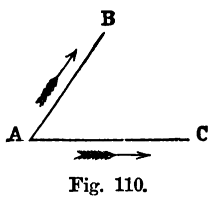


Fig. 110.

Parallelogram of Forces.—The exact direction in which a body acted on by two forces will move, and the exact extent of its motion, is represented by the diagonal of the parallelogram, of which the lines representing the forces are two adjacent sides. (Page 19.)

The Triangle of Forces.—The resultant of two forces may also be found by constructing a triangle, of which

two sides shall represent the given forces; the third side represents the resulting force. (Page 20.)

Resolution of Forces.—A single force may be resolved into any two forces, whose representing lines will form a triangle with a line representing, on the same scale, the single force. Thus, A H may be resolved into A F and A G; also, A F may be resolved into A B and A C, and A G into A D and A E. (Page 24.)

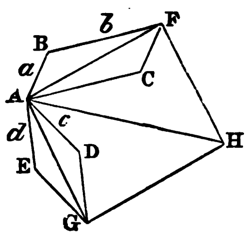


Fig. 111.

Composition of Parellel Forces.—The resultant of two parallel forces acting in the same direction, is parallel to both and between them. If A D and B C be two parallel forces acting towards D and C, the resultant E F is between A D and B C, is equal to their sum, and is parallel to them. (Page 26.)

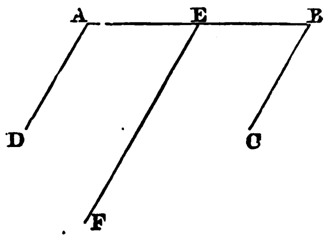


Fig. 112.

A Couple.—A pair of equal parallel forces acting in opposite directions is called a couple. (Page 28.)

The Moment of Force.—The power of a force to produce motion is called its moment. (Page 29.)

Centre of Parallel Forces.—The point at which the whole of any number of parallel forces may be considered to act (*i. e.*, the point at which the resultant acts) is called their centre. Thus, *a* is the centre of parallel forces acting at A and B; *b* is the centre of parallel forces acting at A B and C; *c* is the centre

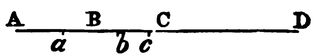


Fig. 113.

of parallel forces acting at A B C and D. (Page 30.)

Work Done by a Force.—The work done by a force is estimated by the number of pounds it can raise, and the number of feet through which it can raise them. (Page 31.)

Unit of Work and Horse-power.—The unit of "work" is the raising of 33,000 lbs through one foot. The power of doing this is called a "horse-power." When time has to be considered, one minute is the unit. The power of one horse is estimated to be equal to the work of raising 33,000 lbs through one foot in one minute. (Page 32.)

Different Conditions of Matter.—Every substance is either solid, liquid, or gaseous, and which it is depends upon the temperature of the body. (Page 32.)

Divisibility.—Every body can be divided into very small particles, either by mechanical means or by heat. (Page 34.)

Compressibility.—Every body can be compressed in a smaller space by having its particles forced into more close proximity. (Page 34.)

Elasticity.—All bodies have, more or less, the property of returning to their original shape when but slightly disturbed. Some bodies, such as steel and india-rubber, are more elastic (*i.e.*, have this property more strongly developed) than others, such as copper or putty. (Page 35.)

Gravitation.—Every solid or liquid substance falls, or tends to fall, towards the centre of the earth. (Page 36.)

Centre of Gravity.—In every solid body there is one point round which, if it be supported, the whole body will balance. This point is called the centre of gravity. (Page 40.)

Position of the Centre of Gravity.—In every regular figure the centre of gravity can be found by calculation. In every figure it can be found by experiment. (Page 41.)

The Lever.—Any two weights will balance about a

fulcrum, if their moments about it be equal. A rigid bar so balanced about one of its points is called a lever. (Page 43.)

Forces not at Right Angles.—The effect upon a lever, of a force not at right angles to it, may be found by drawing a line from the point of action of the force at right angles to the lever, produced if necessary. (Page 47.)

The Steelyard.—The same weight, acting on the long arm of a lever, may be used to balance varying weights, acting at the extremity of the short arm, by being moved nearer to or farther from the fulcrum, as occasion requires. Such a lever and weight is called a steelyard. (Page 52.)

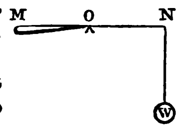


Fig. 114.

The Danish Steelyard.—The weight of one arm of a lever may be made to balance a weight acting at the other arm, by moving the fulcrum. Such a lever, with a movable fulcrum, is called a Danish steelyard. (Page 53.)


Fig. 115.

Equilibrium.—A body will remain as placed (*i.e.*, will be in equilibrium), if the vertical line drawn downwards from its centre of gravity falls within the points of support. Thus a cart-load of hay or of bricks will not fall over so long as the line drawn vertically downwards from the centre of gravity falls between the wheels. (Page 54.)

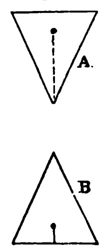


Fig. 116.

Stable and Unstable Equilibrium.—If a body is so placed that a slight disturbance will cause it to fall over, it is said to be in unstable equilibrium, as in the case of a cone balanced on its vertex. If it may be so moved without falling over, its equilibrium is stable, as in the case of a cone standing on its base. (Page 54.)

Laws of the Motion of Falling Bodies.—A body falling freely in a vacuum, passes through 16 feet in the first second,

48 feet in the second second, and 80 feet in the third second. Its velocity at the end of the first second is 32 feet, at the end of the second second 64 feet, and at the end of the third second 96 feet. (Page 56.)

Attwood's Machine is a contrivance for measuring the velocity of a freely falling body. (Page 58.)

Morrin's Machine is a contrivance for recording the direction and velocity of the motion of a freely falling body. (Page 59.)

Motion in a Circle.—A moving body which would move in a straight line, but that it cannot go beyond a given distance from another body, will describe a circle round that body. Thus a body which would travel in the line E B, but for being fastened to A, will move in the circle E C D. (Page 60.)

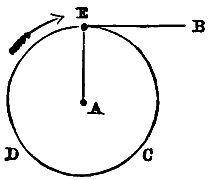


Fig. 117.

The Simple Pendulum.—A pendulum set swinging, will always move to and fro at the same rate. Of two pendulums, the longer will move the more slowly. A simple pendulum, 39·14 inches in length, will vibrate once every second. A simple pendulum is a small weight suspended by a very fine flexible thread. (Page 60.)

The Compound Pendulum.—A weight suspended by a rigid rod is a compound pendulum; the weight may be the weight of the rod itself. A compound pendulum vibrates at the same rate as a simple pendulum two-thirds its length. (Page 61.)

Determination of force of Gravity.—The force of gravitation (*i.e.*, the attraction of the body of the earth for all bodies near it) is determined by the time a pendulum of given length takes to vibrate. If the force be increased, it vibrates more quickly; if reduced, more slowly. The nearer to the centre of the earth the greater is the force. (Page 64.)

Energy.—A falling body derives its force from having

been raised; the greater the height to which it is raised, the greater the force of its fall. (Page 66.)

Pressure transmitted through a Fluid.—Pressure communicated to a fluid is transmitted equally in all directions. (Page 68.)

Centre of Pressure of a Fluid against a plane area.—If a plane surface be subjected to pressure from a fluid, it will be pressed equally on all points, if it be horizontal, and the centre of pressure will be at the centre of the surface; if it be vertical, the lower part will be more pressed than the upper, and the centre of pressure will be below the centre of the surface. (Page 68.)

Pressure of a Fluid on an Immersed Body.—If a body be heavier than an equal volume of water, it will sink in water; if it be lighter it will sink until it has displaced a volume of water equal in weight to itself. (Page 69.)

Specific Gravity.—The specific Gravity of a body is the ratio of its weight to the weight of an equal volume of water. It is determined by weighing it with an equal volume of water. (Page 71.)

The Hydrostatic Balance is an apparatus for ascertaining the specific gravity of a solid, by weighing it first in air and then in water. (Page 71.)

Hydrometers.—An apparatus for ascertaining the specific gravity of a liquid by inspection is called an hydrometer. (Page 73.)

The Specific Gravity Bottle—is an apparatus for ascertaining the specific gravity of a powder. (Page 75.)

Equilibrium of a Floating Body.—For a body to float in a liquid, and to remain upright, its centre of gravity should be below the *meta-centre*, which is the point where the line of the centre of pressure of the liquid meets the line passing vertically through the centre of gravity when the body is upright: that is, G should be below the point where the lines A and B meet. (Page 75.)

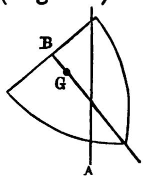


Fig. 118.

Capillary Elevation and Depression.—A liquid will rise inside a tube of any material to which it will adhere, and will rise higher the smaller the diameter. If the liquid does not adhere to the tube it will be depressed, and be lower the smaller the diameter. (Page 76.)

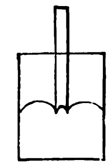


Fig. 119.

Proof that air is a heavy elastic fluid.—Air is proved to be a fluid by the readiness with which it fills up all openings. That it is elastic is shown by its filling any space, however large, and by its expanding under a reduced pressure. (Page 78.)

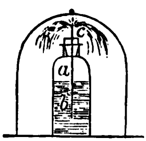


Fig. 120.

Toricelli's Experiment was one to test whether the limit of 32 feet to the rise of water in a pump was owing to the pressure of the air. (Page 79.)

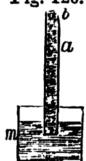


Fig. 121.

The Cistern Barometer.—A column of mercury is kept in a tube by the pressure of the air, and rises and falls as the pressure increases or diminishes. The lower, and open end, is immersed in a small cistern of mercury, whence the name. (Page 79.)

The Aneroid Barometer is a small metal box, from which the air has been pumped out. The lid rises and falls as the pressure of the external air varies, and this very small movement is magnified by a system of levers. (Page 80).

Boyle's Law.—The volume of a gas varies inversely as the pressure upon it (page 83). But this law is limited by certain conditions. (Page 84.)

Compressed Air Manometer.—An apparatus for measuring the pressure of a gas. (Page 84.)

The Suction Pump.—If an empty tube be placed with one open end in water, the water will. rise in it to a height of about thirty feet. A machine for pumping

water by means of an empty tube is called a suction pump. (Page 85.)

Hydraulic Press.—Water being almost incompressible, cannot be made to occupy less space. If, therefore, water be pumped into a cylinder already full, in which there is a piston, the latter will be forced out by the water, whatever be the external pressure upon it. (Page 86.)

The Lever.—Two unequal weights will balance upon a lever with equal arms if they act at different angles, so that their effective forces are equal. (Page 88).

The Screw is an inclined plane, wrapped round a cylinder. (Page 90.)

The Wedge is formed by two inclined planes, having their bases joined. (Page 91.)

The Pulley is a machine for raising heavy bodies, and at the same time supporting the greater portion of their weight. (Page 92.)

The Resultant of Parallel Forces.—The resultant of two parallel forces is parallel to them, acts between them, and is equal to their sum or their difference. (Page 96.)

The Differential Axis is a machine for raising weights by means of one rope wound round two axles of different sizes. (Page 102.)

Compound Levers.—A series of levers, connected so as to transmit force from one extreme to the other, is called a compound lever. (Page 105.)

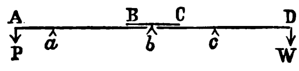


Fig. 122.

Friction.—The motion of any one body over another is always more or less retarded by the roughness of the surfaces in contact. This roughness, and the consequent hindrance to motion, are called friction. (Page 106.)

Motion.—The transition or transference of a body from one point in space to another is called motion. (Page 107.)

First Law of Motion.—A body at rest remains at rest,

and a body in motion continues in motion, until acted upon by some external force. (Page 110.)

Second Law of Motion.—Every force acting on a body, whether at rest or in motion, produces its full effect of motion. (Page 114.)

PROBLEMS AND SOLUTIONS.

1. *A lever, 10 feet long, has a weight of 6 lbs. at one extremity: the fulcrum is 1 inch from the same end. What weight at the other end will the 6 lbs. balance?*

Here A C = 1 inch, A B = 10 feet, and W = 6 lbs. Required the value of P. The fulcrum, C, being very near one end, A, of the lever, a very small weight at the end of the long arm, B C (in this case 9 feet 11 inches long), will balance W.

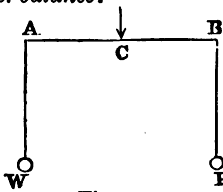


Fig. 123.

As B C : A C : : W : P—
i. e., as 119 inches : 1 inch : : 6 lbs to P

$$\therefore P = \frac{1 \times 6}{119} \text{lbs} = 6 \text{ lbs divided by } 119 = 12{\cdot}8 \text{ drams.}$$

2. *The radius of an axle is 4 inches; of the wheel 3 feet. If a weight of 10 lbs. be hung from the axle, what weight must be suspended from the wheel to balance it? and what will be the whole pressure on the axis of the machine?*

The 10 lbs weight suspended from the axle tends to turn it to the right (if hung as in fig. 124), and acts at the end of a lever 4 inches in length; for the centre of the axle, which is the only fixed point, is the fulcrum. The weight suspended from the wheel on the opposite side (to the left in the fig. 124) acts at the end of a lever 36 inches in length.

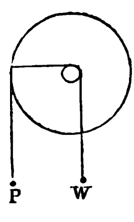


Fig. 124.

So that as

$$\begin{array}{ccccccc} & \text{Radius of Wheel} & : & \text{Radius of Axle} & :: & \text{W} & \text{P.} \\ & \text{In.} & & \text{In.} & & \text{lbs.} & \\ \therefore & 36 & : & 4 & :: & 10 & : \text{P} \end{array}$$

$$\therefore \quad \frac{4 \times 10}{36} = \text{P} = \frac{40}{36} = 1\tfrac{1}{9} \text{ lb.}$$

—*i. e.*, $1\frac{1}{9}$ lb will at the edge of the wheel balance 10 lbs acting on the axle. Secondly, as to the whole pressure on the axle, the weights of W and P together, added to the weight of the wheel and axle, have the centre of the axle as the centre of gravity, and therefore can be supported at that point by an equal upward pressure. Without counting the weight of the wheel and axle, the pressure is $W + P = 10 + 1\frac{1}{9}$ lbs $= 11\frac{1}{9}$ lbs.

3. *What is the relation of power to weight in the screw? Show how the inclination of the thread of the screw influences the power of this machine.*

A screw is an inclined plane wrapped round an axis. When a weight is forced up by a screw, it is really that the inclined plane is forced under it, so as to raise it somewhat after the manner of a wedge. Now, the ratio of power required to raise a given weight is the ratio of the height of the plane to its length, if the power act along the plane; or the ratio of the height of the plane to the base, if the power act horizontally. In the ordinary action of the screw, as when heavy weights are raised by means of levers by which the screw is turned, the power is usually applied horizontally. Therefore, if H be the height of one thread of the screw, and C be the circumference of the circle through which the handle of the lever turns, we have

$$P : W :: H : C$$

i. e., the power required to raise a given weight is as the distance through which the weight is raised is to the distance through which the applied force moves. This is the old theorem over again, however much disguised.

One pound moving through ten feet will raise ten pounds through one foot, not counting the loss ot power necessary to move the machinery.

The labour of raising a weight directly from C to B is exactly the same as that of raising it from A to B. The distance from A to B is greater than that from C to B; but the weight to be raised has to be entirely supported throughout the short distance, while when passing from A to B it is partly supported by the plane on which it rests. But in raising a weight from A B there is also horizontal motion. So that the labour of moving a weight from A to B is equal to the labour of moving it from A to C and then to B.

Fig. 125.

If a piece of paper, the shape of A B C, be wrapped round a penholder, we have a model of a screw, which shows at once its relation to an inclined plane. At the western extremity of the Undercliff, in the Isle of Wight, there is a light-house which has a single stair-case winding round and round, inside, until it reaches the top. This may be called an inclined plane or a screw. In the same way many a mountain pass is crossed by means of a road that winds to and fro, until it is long enough to reach the top, without being too steep for a carriage-road. Each of these is an inclined plane; but is not a screw, since it does not wind round any given axis? The second part of our question is, therefore, to be answered by saying, that the more steep an inclined plane, or the greater the inclination of the thread of a screw, the greater will be the labour required to raise any given weight that distance.

4. *A rectangular slab (specific gravity* 3) 1 *inch thick,* 10 *inches long,* 8 *inches broad, is placed on a talle, with one side (whose edge is parallel with the edge of the table) projecting beyond it: a weight of* 1 *lb. is placed on the*

opposite edge. How far can the slab be pushed off the table without falling?

In fig. 126, A is the slab, B the table, and w the 1 lb weight. The question is, how much of the slab will balance the remainder and the pound weight? Were there no weight, w, the slab would evidently just balance when half off the table, since the edge of the table would then support its centre of gravity, c. But more of the slab must be pushed off to balance the weight, w; and it will just balance when the weight of the outer portion, acting at its centre of gravity, balances the inner portion, and the weight acting at their common centre of gravity.

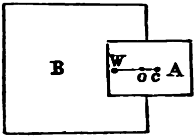


Fig. 126.

The centre of gravity of the piece of marble may be assumed to be at its middle point, which is 5 inches from the end and 4 inches from the side. This is joined to that of the weight, whose centre of gravity acts on the edge of the piece of marble.

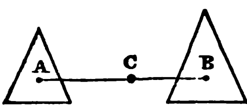


Fig. 127.

The centre of gravity of the two, marble and weight, is to be found in the line joining their respective centres of gravity, in the line joining w and c, after the manner described in page 42. This is equally the correct method, whether the bodies are entirely separated, except by the connecting-rod (as in fig. 127), or whether one is upon the other, as with the weight and the marble, as here (fig. 126).

Now, the weight of the marble may be found by comparing its volume with that of a cubic foot of water, which may be taken as weighing 1,000 ounces, and taking into account its specific gravity as compared with that of water.

The size of the marble is $10 \times 8 \times 1 = 80$ cubic inches:

the number of cubic inches in a cubic foot is $12 \times 12 \times 12 = 1{,}728$.

Therefore, as $\overset{\text{In.}}{1{,}728} : \overset{\text{in.}}{80} :: \overset{\text{oz.}}{1{,}000}$: weight of 80 cubic inches of water $= 46$ oz.

But the marble is three times as heavy as water, and therefore it weighs $46 \times 3 = 138$ oz. The pound weight weighs 16 ounces, and the two centres of gravity are 5 inches apart. The total weight of both is $138 + 16 = 154$ oz.

Therefore, $138 : 16 :: 5$ inches: distance of *o* from *c* (fig. 126).

The answer is very nearly $\cdot 6 =$ about half an inch; so that the addition of the pound weight moves the centre of gravity $\frac{1}{2}$ inch; and the piece of marble may be pushed $5\frac{1}{2}$ inches off the table, leaving $4\frac{1}{2}$ inches on it—the additional inch being balanced by the pound weight.

I have assumed the marble to be pushed off the table endways—it might be pushed off sideways—and the answer would then be found in precisely the same way, excepting that the final statement would be—

$$\text{As } 138 : 16 :: 4 : \cdot 4,$$

and the marble could be pushed off only $\frac{4}{10}$ of an inch instead of $\frac{5}{10}$, as before.

5. *If a body be let drop from a height of* 1,000 *yards, how long will it be before it strikes the ground (disregarding resistance of the air)?*

$$s = \tfrac{1}{2} g t^2 —$$

that is, the space = half the square of the time multiplied by the effect of gravity. But this may be written—

$$2s = gt^2$$

$$\therefore \frac{2s}{g} = t^2$$

$$\therefore t = \sqrt{\frac{2s}{g}} = \sqrt{\frac{2{,}000}{32}}$$

$$= \sqrt{62 \cdot 5} = 7 \cdot 9 \text{ seconds.}$$

In this problem,—$s = 1{,}000$ yds.
$t =$ the unknown quantity.
$g = 32$ ft.

6. *A cube is placed on a rough inclined plane: through what angle can the plane be raised without causing the cube to topple over?*

The centre of gravity, *c*, of the cube will be at its centre, supposing it to be of equal substance throughout. If the table be slowly and regularly raised on one side, the centre of gravity will remain between the table and the vertical line drawn through *o*, the point about which the cube would move in toppling over. Until the table be raised to an angle of 45° the cube would not move (excepting to slide down the inclined plane, which its roughness would tend to prevent). At 45° the centre of gravity would be exactly above *o*, and the cube would fall from the table, turning over upon *o*, the moment the inclination became greater than 45°. But if the table were raised quickly, the impetus given to the cube would probably throw it over before it reached 45°.

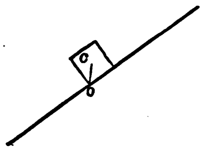


Fig. 128.

7. *What is meant by the pressure of a fluid against a surface immersed in it,* e. g., *against the flood-gate of a lock? How is it measured? and in what line does the resultant power act?*

Let the lock gate be supposed to be divided horizontally into a great number of very narrow pieces, *a*, *b*, *c*, &c. Then the pressure against *b* will be greater than against *a*; for the water pressing against *b* has to support the weight of *a*, the water above it, and is therefore forced against the gate and the banks of the river or canal with the extra force due to this. For the same reason, the water

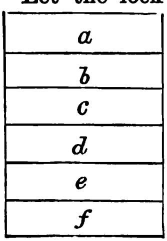


Fig. 129.

opposite to c is pressed down, and also sideways, with still more force, and the water opposite d e and f with even more.

But for this increase of pressure for each lower stratum of water, the pressure would be alike at every point of the lock gate, and the centre of pressure would therefore be at the centre of the gate—*i.e.*, an equal force at this central point outside the gate would exactly counterbalance the whole pressure of the water upon the entire surface of the gate.

But evidently this centre of pressure will be lower than the centre of the gate, owing to the greater pressure upon the lower portion of the water. But, also, it will be in the line passing vertically through this centre, as there will be equal pressure on each side of the lock gate.

Therefore the centre of pressure will be in the vertical line passing through the centre of the lock gate, and at a distance below it, depending on the relation between the weight of the water and the pressure it exerts from some other cause.

8. *Explain the mechanical difference between air and water as regards pressure and compressibility.*

I pour some mercury into the bottom of a curved tube (fig. 130), up to the point o, and securely fasten the end, b, by a screw lid. The air in o b occupies the same space as if outside the tube. Now, I pour mercury into the long arm, n, until it is 30 inches higher than o: the air in o b will be compressed into half its former space. I pour more mercury into n, until it is 30 inches higher still (*i.e.*, 60 inches higher than o); the air in o b is now compressed into one-third its former space.

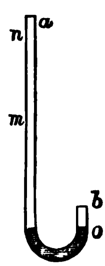


Fig. 130.

I repeat the experiment, having water in o b instead of air, and I find that it is not compressed into a smaller space, but con-

tinues to occupy the whole space *o b*. Therefore I say that air is compressible—that is, is compressed into a smaller space; and that water is not compressible. (This is not quite true, because water can be made to occupy a little less space, but only by very great pressure.)

Secondly, I put a bottle partly filled with water and tightly corked, with an open tube down the middle of the cork into the water, under the receiver of an air-pump, and exhaust the air. The pressure of the air forces the water out through the tube (page 78). But if the bottle had been quite full of water, the water would not have been forced out, though the water in the space *a* would be heavier than the air. But air is elastic, and is kept at any given bulk only by pressure, expanding the moment that pressure is reduced. Water is not elastic, and continues to occupy the same space, even when the pressure of the air upon it is withdrawn.

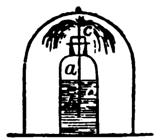


Fig. 131.

9. *How is it that ships can be made of material heavier than water? Why do ships carry heavy ballast in the hold?*

A lump of iron sinks in water; a flat thin piece of iron, or a fine needle, laid carefully on the surface of water, will float; a hollow vessel of thin iron will float freely upon water, if the opening be closed so that the water cannot enter. In this case the total weight of the iron and contained air is less than an equal volume of water (the lightness of the air counterbalancing the heaviness of the iron), and therefore the vessel of iron floats. An iron ship is such a hollow vessel of iron. Notwithstanding the enormous weight of an iron vessel, it is still less than that of an equal volume of water, because of the large empty space within it.

Vessels are laden with heavy ballast in the hold, so

that the centre of gravity of the whole may be low down; and therefore the vessel will be less likely to be overturned by a storm (page 76).

10. *If* 16 *ounces balance* 20 *ounces at the ends of a lever* 8 *ft. long, where is the fulcrum (neglecting the weight of the lever)?*

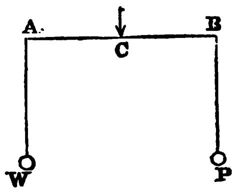


Fig. 132.

The lengths of the arms of the lever must be inversely as the weights. If $W = 16$, and $P = 20$, then

$$A\,C : B\,C :: 20 : 16\text{—}$$

i.e., $A\,C : B\,C :: 5 : 4$;

then $A\,C = \frac{5}{9}$ of 96 inches,
and $B\,C = \frac{4}{9}$ of 96 inches,
$\therefore A\,C = 53\frac{1}{3}$ inches, and $B\,C = 42\frac{2}{3}$ inches.

11. *A weight of* 60 *lbs is supported on an inclined plane by a weight of* 30 *lbs: what is the relation of the height of the plane to its length (the weight of* 30 *lbs hangs vertically: a string from it passes over a pulley at the vertex of the inclined plane, and is parallel to the inclined plane till it reaches the weight of* 60 *lbs)?*

The weight $P = 30$ lbs balances the weight $W = 60$ lbs, but one acts vertically at A, the other at an angle, M B N. The whole 30 lbs of P acts at A, but only a portion of W at B. By drawing the lines B M and M N, completing the triangle, B M N, right angled at M, we get the relative amounts of the forces acting along B M, M N, and B N. Of these B M is the only force that counterbalances P; and of this triangle we know that

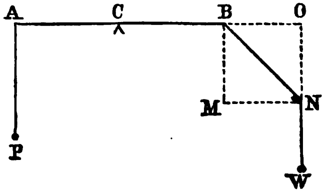


Fig. 133.

B N represents 60 lbs, the whole weight of W, and B M represents 30 lbs; for since it balances P, it must be equal to it. But B N is the length of the plane, and B M its height; therefore its height is half its length.

12. *Explain the method of measuring the pressure of the atmosphere.*

To weigh a piece of stone or metal (or in fact anything), we have to find an equivalent weight in some other substance taken as a standard. For ordinary purposes we speak of pounds, ounces, &c., and weights are usually made of iron or brass; but if all the weights in use required verification, it would be necessary to find what was the arbitrary standard with which to verify them. A certain quantity of pure water at a given temperature and pressure is one such standard; and the force with which the earth draws that down is taken as the unit of weight.

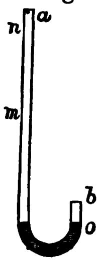


Fig. 134.

In the same way, to measure the pressure of the atmosphere we must have a standard of comparison. Any one will do, but the most convenient is mercury, because of its remaining both liquid and pure under most ordinary circumstances, and also because, it being heavy, a small quantity of it suffices for the purpose.

We therefore put the air in one scale and the mercury in another—that is, we put them so that they act against each other. The means of balancing these are found in the use of a bent tube (fig. 134), at the bottom of which is a little mercury serving as the beam of the scales. The weight of mercury in the tube *a* pushes this one way, and the weight of the air in the tube *b* pushes it the other. When these two forces are equal, the mercury is level with *o* in the tube *b*, and the amount of mercury in the tube *a* above this level shows the pressure of air.

13. *Draw two forces,* A B *and* B C, *at right angles to each other. Suppose that a force of* 10 *lbs acts at* A, *from* A *to* B, *and a force of* 12 *lbs on the point* A, *from* A *to* C; *find by a construction made to scale the magnitude and direction of the force which, acting at* A, *would balance these forces.*

The lines A B and A C (which should be at right angles) are supposed to be drawn of indefinite length; we must therefore cut off from A, along A B, some length to represent the force of 10 lbs, and then a length from A, along A C, to represent the force of 12 lbs.

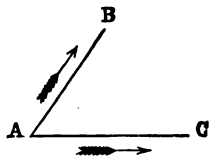


Fig. 135.

The parallelogram being completed, its diagonal, A D, represents the magnitude of the resultant according to the same scale as used in drawing A B and A C to represent the forces; and the same line also represents the direction of the force that, acting at A, would balance the two forces; but this direction is D A, not A D.

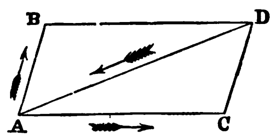


Fig. 136.

14. *A bricklayer's labourer with his hod weighs* 170 *lbs; he puts into the hod* 20 *bricks, weighing* 7 *lbs each, and then carries them up a ladder to a height of* 30 *feet, How many units of work does he do? and if he can do* 1,500,000 *units of work per day, how many bricks will he take up the ladder in a day?*

The man, the hod, and 20 bricks weigh altogether $170 + 20 \times 7 = 310$ lbs, and this raised to a height of 30

feet = 9,300 lbs raised one foot. One unit of work is one pound raised one foot; therefore he performs 9,300 such units in ascending the ladder to a height of 30 feet with his load of 20 bricks.

Now $\frac{1,500,000}{93 \cdot 00} = 161 \cdot 2$. Therefore he would have done 1 day's work = 1,500,000 units of work, when he had ascended the ladder 161·2 times. Each time he would carry up 20 bricks: therefore the total number he would carry is $161 \cdot 2 \times 20 = 3,224$. He would likewise raise his hod and himself 161·2 times; also, no account is taken of the labour of coming down after each ascent.

15. *A cube whose side is* 18 *inches is placed in water, so that the top face is level with the surface. Find the pressure on its lowest face, one cubic foot of water being reckoned to weigh* 1,000 *ounces.*

The cube measures altogether $3\frac{3}{8}$ cubic feet, and has therefore displaced $3\frac{3}{8}$ cubic feet of water, which weigh 3,375 ounces. This displaced water is only prevented from returning to its place by the weight of the cube; and therefore these two pressures must be equal.

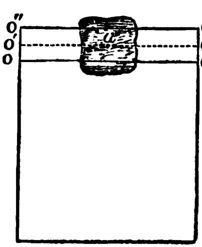


Fig. 137.

That is, the weight of a is balanced by the weight of the displaced water between 0 and 0'', and each of these weights is 3,375 ounces. But this tends to force up the cube a, and therefore acts on its under surface, which measures 18×18 square inches = 324 square inches; and therefore the pressure on this lower surface is 10·4 ounces on each square inch, or 219 lbs on the whole surface.

16. *State exactly what is meant when it is said that the accelerating force of gravity near the earth's surface is very nearly* 32·2 *feet per second?*

A body let fall from any height commences its descent immediately it is free to move. At the beginning of the first second, its velocity equals 0, for it is only then that it begins to move. At the end of the first second, its velocity is 32·2 feet per second—that is, it is at that precise point of time moving at that rate.

This is also its rate of motion at the beginning of the second second (for the end of the first is the beginning of the second); but at the end of the second its velocity is 64·4 feet per second.

It will be seen by this Table that the velocity of a freely falling body is increased by 32·2 feet every second: and this is the result of the accelerating force of gravity; or rather the acceleration is the result of the force of gravity, which acts continually, and so tends to draw a falling body more and more rapidly.

Velocity at the		
End of	Ft. per Second.	Beginning of
	0	1st Second
1st Second	32·2	2nd „
2nd „	64·4	3rd „
3rd „	96·6	4th „
4th „	128·8	5th „
5th „	161·0	6th „
6th „	193·2	7th „

Or it may be put more clearly still in another way:—

Any two bodies have a mutual attraction, and the force of this attraction is greater the nearer are the bodies. A falling body is continually approaching nearer to the earth, and therefore (the distance being lessened) the force of gravitation acts more forcibly, and produces more rapid motion.

"The accelerating force of gravity" is not a clear expression: gravity has no accelerating force; but the acceleration of speed is a result of the force being brought into increased operation.

17. *A body falls freely, under the action of gravity from rest, for* 6 *seconds. What is the space passed through during the last* 2 *seconds of its motion?*

Every second the velocity increases 32·2 feet per second, and the actual distance traversed is 16·1 feet more than if gravitation had ceased to act at the end of the preceding second:— Thus, a falling body would fall through 64 feet in the 3rd second, from its motion at the beginning of that space of time; but its actual fall is increased 16·1 feet, and its velocity is increased 32·2 feet by the continued motion of gravity.

Seconds.	Rate of Motion at beginning of Second.	Increase due to Gravitation.	Total Space passed through.
1st	0	16·1	16·1
2nd	32·2	16·1	48·3
3rd	64·4	16·1	80·5
4th	96·6	16·1	112·7
5th	128·8	16·1	144·9
6th	161·0	16·1	177·1

In the last two seconds, the falling body would pass through 322 feet = 144·9 (during the 5th second) + 177·1 (during the 6th second).

18. *Show that when two couples are in the same plane, they will be in equilibrium, if the moments are equal and of contrary signs.*

The two forces, A and B, tend to turn the rod, X Y, in one direction; while the two forces, C D, tend to turn it in just the opposite direction. The effective force of each couple is called its moment. If the moments of these two couples are equal, the rod, X Y, will evidently not be moved at all. This opposite action of contrary couples is usually expressed by calling one + and the other −.

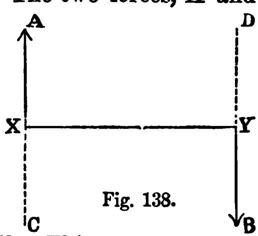


Fig. 138.

If, therefore, the moment of A B = $+m$, and the moment of C D = $-m$, evidently the result is expressed by the equation $+m - m = 0$.

19. *Draw two straight lines,* A B, A C, *containing an angle of* 60°; *let a force of* 10 *lbs act from* A *to* B, *and one of* 20 *lbs from* A *to* C. *Find, by construction, or otherwise, the magnitude of their resultant.*

Since A C represents 20 lbs, and A B only 10 lbs, A C = 2 A B. I therefore draw the parallelogram A D, so that A C is twice A B; then the diagonal, A D, represents the resultant.

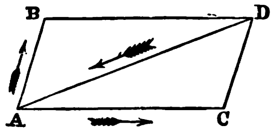


Fig. 139.

If I measure this, and compare its length with the length of A B or A C, I have ascertained the magnitude of the resultant by construction. Thus:—

As A B : 10 : : A D : resultant = 26·4 (nearly).

So that if A B represent the distance through which a weight of 10 lbs would move a body, and A C twice that distance, then A D represents the distance through which the same body would be moved from A, either by A B and A C combined, or by a single force of 26·4 lbs acting at A along A D.

But the length of A D can be ascertained by calculation, thus:—

If the angle B A C be 60°, then also the angle D C E is 60°, C D E is 30°, and C E is half of D C = 5, so that A E = A C + C E = 20 + 5 = 25, and $AD^2 = AE^2 + DE^2 = 25^2 + 8{\cdot}5^2$, and $AD = \sqrt{697{\cdot}25} = 26{\cdot}4$.

This calculation depends upon the 47th proposition of the 1st Book of Euclid's Geometry, and the point E is found by producing A C to E to meet a line from D perpendicular to A C.

20. *A rod of uniform section and density weighing* 12 *lbs rests on two points on the same horizontal line, one under each end. What pressure is there on each point?*

The centre of gravity, G, is at the centre of the rod

equi-distant from each point of support. Therefore the weight, 12 lbs, will be supported equally by these two points, A and B, and each has to bear 6 lbs. It might also be considered thus:—

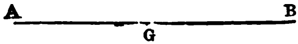


Fig. 140.

The pressure of the weight of A B at G would cause the bar to wind round A and hang from it if B were away, or from B if A were away. Therefore since the presence of B prevents this, we may calculate it thus:—

The moment of 12 lbs acting at G at the distance A G from A is twice the moment of the upward force exerted by B at twice the distance from A. Therefore the force exerted by B equal to the pressure upon it, is 6 lbs. In the same way the pressure upon A is also 6 lbs, since acting at the whole distance, A B from A, it equals 12 lbs acting at half that distance.

21. *A square board weighs 4 lbs, and a weight of 2 lbs is placed at one of its corners. Show by a figure the position of the centre of gravity of the board and weight.*

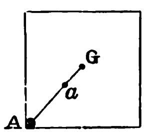


Fig. 141.

The centre of gravity of the board is at its centre, G, and the weight at the corner, A, will have the result of moving the centre of gravity from G to a, which point is to be found by taking one-third of the distance, G A from G.

For the centre of gravity of two bodies, A and B is at C in the line joining their centres of gravity, nearer to the heavier of the two, so that the moments of the two are equal. Now, the distance G a must be to A a as A is to G, so that G a : A a : : 1 : 2 $\therefore$ G $a = \frac{1}{3}$ of G A.

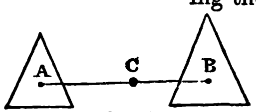


Fig 142.

22. *A cup when empty weighs* 6 *ounces; when full of water it weighs* 16 *ounces; when full of petroleum it weighs* $14\frac{3}{4}$ *ounces. What is the specific gravity of petroleum?*

If the cup and water together weigh 16 ounces, and the cup alone 6 ounces, then the water by itself weighs 10 ounces; in the same way the weight of the petroleum is $8\frac{3}{4}$ ounces.

Therefore the specific gravity of the petroleum is to that of the water as $8\frac{3}{4}$ is to $10 = \frac{8\frac{3}{4}}{10} = \frac{35}{40} = \frac{7}{8} = \cdot 875$.

23. *How many gallons of water would a steam engine of* 10-*horse power raise in one hour from a depth of* 200 *fathoms?* (*A gallon of water weighs* 10 *lbs.*)

A "horse-power" is defined (page 32) to be the power of raising 33,000 lbs one foot per minute; therefore 10 horse-power could raise ten times this, or 330,000 lbs. Now 330,000 lbs per minute becomes 19,800,000 per hour. Also 200 fathoms are 1,200 feet, and 19,800,000, divided by 1,200 = 16,500, which is the number of gallons the given horse-power would raise from a depth of 200 fathoms in one hour, supposing all the power to be available for raising water, and none of it to be lost in working the machinery.

24. *How is the velocity of a moving body measured when uniform? If a velocity is denoted by* 48 *when feet and seconds are taken as units, what number will denote the same velocity when the units are miles and minutes?*

A velocity of 48 would mean that the body was moving at the rate of 48 feet per second. Now a second is $\frac{1}{60}$ of a minute, and a foot is $\frac{1}{5280}$ of a mile. Therefore 48 feet is $\frac{48}{5280} = \frac{1}{110}$ of a mile, and 48 feet per second is $\frac{48}{5280} = \frac{1}{110}$ of a mile in $\frac{1}{60}$ of a minute, and therefore

$\frac{60}{110}=\frac{6}{11}$ in a minute. So that a velocity of 48 feet per second may also be described as being a velocity of $\frac{6}{11}$ of a mile n a minute.

25. *A rod of uniform section and density weighs* 10 *lbs.; a weight of* 10 *lbs is tied to one end, and a weight of* 20 *lbs to the other end. Under what point of the rod must the fulcrum be placed for equilibrium*

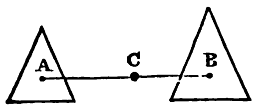


Fig. 143.

If the weight of the rod be not counted, the point round which A and B will balance is found by the usual method. Thus, if A be 10 lbs, and B be 20 lbs, then A C : B C : : 20 : 10., *i.e.*, A C will be twice B C, and therefore A C will be two-thirds of A B, measured from A. But now we have a third weight, 10 lbs, the weight of the rod; this acts at D, the centre point of A C. Therefore we have 30 lbs (A+B) acting at C, and 10 lbs (the weight of the rod) acting at D. The centre of gravity of these two will be at E, between D and C. For D E : C E : : 30 : 10, $\therefore$ D E = 3 times C E and C E is one quarter of C D from C. Round this point the bar and the two weights all balance, it being the centre of gravity of the whole.

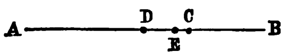


Fig. 144.

26. *An air-pump is so constructed that one-third of the contents of the receiver is removed at every stroke. At the*

commencement the pressure of air is 30 *inches. What is it after the third stroke?*

The first stroke removes $\frac{1}{3}$ of the air, and as the pressure is proportional to the density, that is reduced from 30 inches to 20 inches, *i.e.*, the pressure had been enough to balance that of 30 inches of mercury, it is now only enough to balance 20 inches. The second stroke removes a third of the remaining air, and the pressure is again reduced one-third, and is now equal to $\frac{2}{3}$ of 20 inches $= 13\frac{1}{3}$ inches. The third stroke removes a third of the air still remaining, and the pressure is now reduced to $\frac{2}{3}$ of $13\frac{1}{3}$ inches $= 9$ inches nearly.

It might be at first thought that the whole of the air would be exhausted by three strokes, if each stroke removed one-third; but it is one-third of the air present at the time of the stroke, not one-third of the whole of that at first present, that is exhausted at each stroke, so that the air would never be entirely exhausted by this method, since two-thirds of the air present at the commencement of each stroke would remain after it.

27. *If a small heavy ball be suspended by a fine thread* 12 *feet long, how many small oscillations will it make in one minute?*

The time of vibration of a simple pendulum is found from the equation, $T = 3{\cdot}1416\sqrt{\frac{\text{Length of pendulum}}{\text{Force of gravity}}}$.

(See page 65.) This becomes

$$T = 3{\cdot}1416\sqrt{\frac{12}{32{\cdot}2}}\text{ feet.}$$

$$= 3{\cdot}1416\sqrt{\frac{144}{386{\cdot}4}}\text{ inches.}$$

$$= \frac{3{\cdot}1416 \times 12}{19{\cdot}6}\text{ inches.}$$

$$= 1{\cdot}923\text{ second.}$$

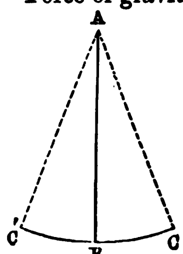


Fig. 145.

So that the given pendulum, 12 feet in length, takes nearly

2 seconds to make one oscillation from C to C′. A pendulum 3·2616 feet long vibrates one per second, and once four times that length (= 13·0464 feet) would vibrate one every two seconds. The given pendulum, being only 12 feet in length, vibrates a little more rapidly—once every 1·923 second. The number of vibrations in one minute will equal 60 divided by $1{\cdot}923 = \frac{60}{1{\cdot}923} = 31{\cdot}2$.

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